Universality in Quantum Liquids and the Lattice Field Theory Problem

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Workshop on Frontiers with Ultracold Gases, Mahón

Outline of the talk

- Quantum Liquids
- Lattice EFT
- Outlook

Quantum Liquids

Question: Given two N-body quantum systems with identical N-body binding energies for all $N \geq N_c$. Do they share thermodynamic properties in the ground state?

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Answer:

- Their Equations of State *must* be different because of normalisation differences at long distances (residue at pole of T-matrix) affecting saturation densities.
- Their equilibrium energy per particle are identical *by construction*.

Quantum Liquids

Two different models with their binding energies adjusted:

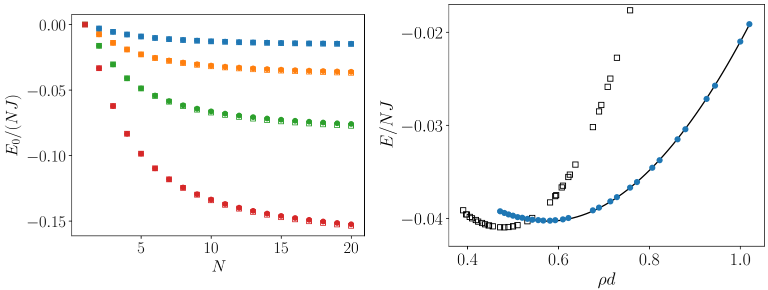
$$H_{U-V} = -J\sum_i b_{i+1}^\dagger b_i + \mathrm{H.c.} + \frac{U}{2}\sum_i n_i(n_i-1) - |V|\sum_i n_i n_{i+1} + 2JN.$$

$$H_{U_2-W} = -J\sum_i b_{i+1}^\dagger b_i + \mathrm{H.c.} - \frac{|U_2|}{2}\sum_i n_i(n_i-1) + \frac{W}{3!}\sum_i n_i(n_i-1)(n_i-1) + 2JN.$$

Quantum Liquids

Question: Given two N-body quantum systems with identical N-body binding energies for all $N \geq N_c$. Do they share thermodynamic properties in the ground state?

Answer: [Morera, Juliá-Díaz, Valiente, PRR **4**, L042024 (2022)]



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Answer II:

- Their Equations of State *must* be different, **but only up to a rescaling of the density and up to exponentially small corrections**.
- Their equilibrium energy per particle are identical *by construction*.
- **What is universal is E/N as a function of the chemical potential μ.**

Quantum Liquids

Equation of State (EoS)

$$\frac{E}{N} = \mu_0 + \beta_2(\rho - \rho_0)^2 + \ldots$$

Liquid drop model – *in vacuo* binding energies

$$\overline{\mu}(N) = \mu_0 + \delta\mu_0 N^{-1/d} + \ldots$$

Central droplet densities (smoothened):

$$\rho(N) = \rho_0 + \delta\rho_0 N^{-1/d} + \ldots$$

Quantum Liquids

Up to exponential corrections $\mu(\rho) = \overline{\mu}(N)$, so that:

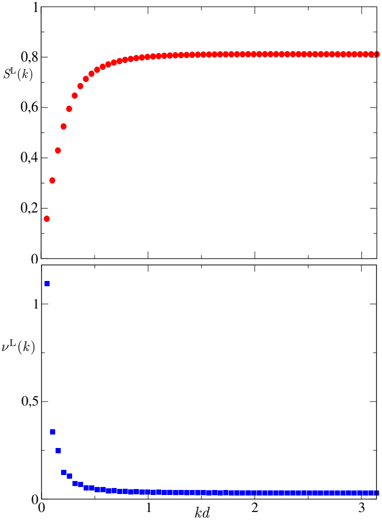
$$mc^2 = \rho_0 \lim_{N \to \infty} \frac{\overline{\mu}'(N)}{\rho'(N)}.$$

This implies that the EoS of two systems with identical binding energies for all N, up to exponentially small corrections, satisfy

$$e_1(\rho) = e_2\left(\frac{\rho_0^{(2)}}{\rho_0^{(1)}}\rho\right)$$

Lattice EFT

The formalism above won't work for Lattice systems, since:



This problem requires

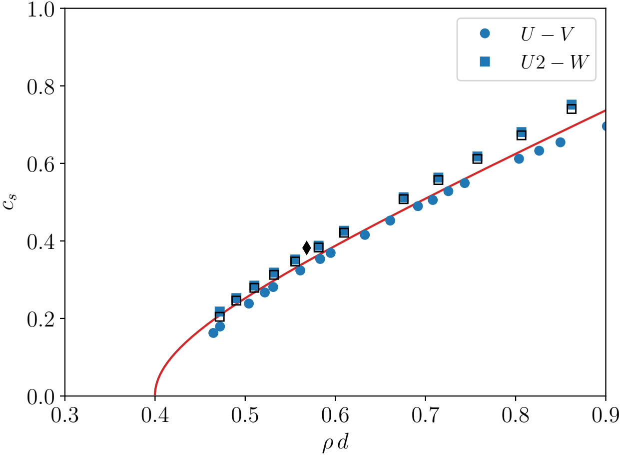
(1) Regularisation-renormalisation

(2) "Negative norms" (non-renormalisability)

(3) Short-range and long-range/low-energy universalities in combination.

Lattice EFT

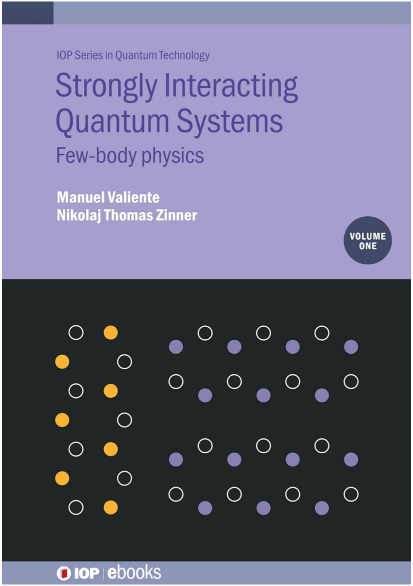
When fixed appropriately, the two supposedly equivalent lattice models give



Outlook

- Lots of room for the development of universal/non-perturbative results in quantum liquids
- Integrable liquids with not-so-unrealistic three-body interactions may be possible to obtain (in progress)
- Non-renormalisable Lattice theories can be fixed to reproduce continuum results

Books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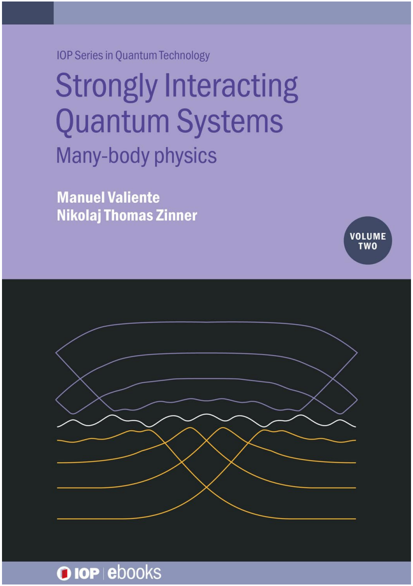
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