Saturating interaction in coherently coupled two-component Bose-Einstein condensates

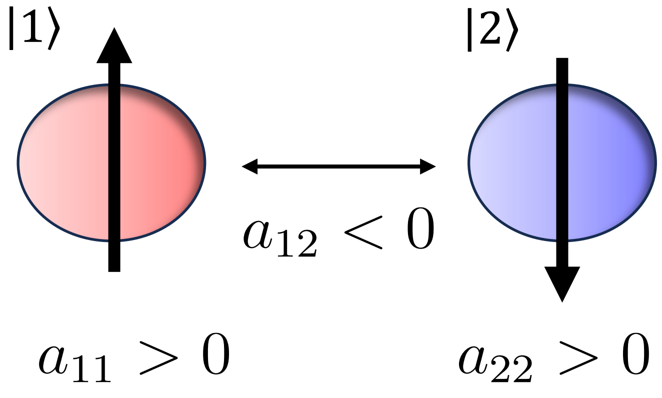
S. Tiengo (3rd y PhD), M. Lévy (1st y PhD), N. Honorè (1st y PhD)
Thomas Bourdel

Research group: Quantum gases with tunable interactions

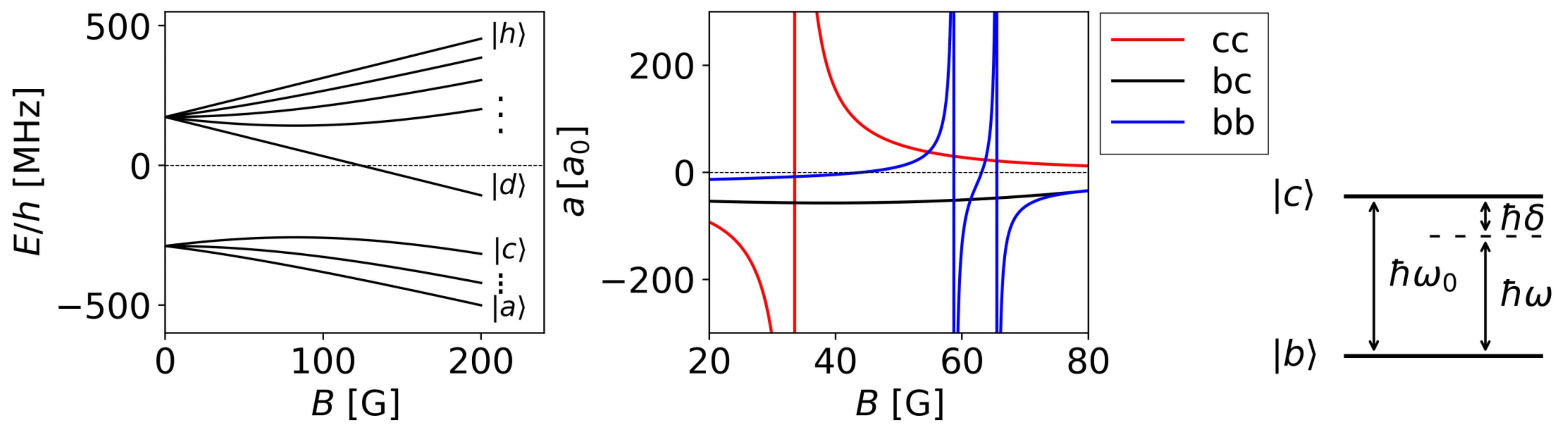
Workshop and School on
“Frontiers in ultracold quantum gases”
10 - 12 June 2026

BEC MIXTURES

Balance inter- and intra-species interactions: higher order interactions effects

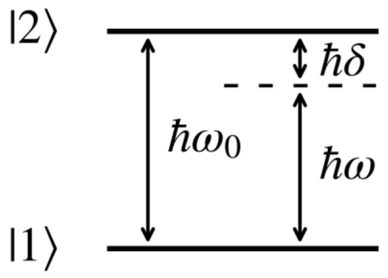


Mixture of ^{39}K atoms with radio-frequency coupling

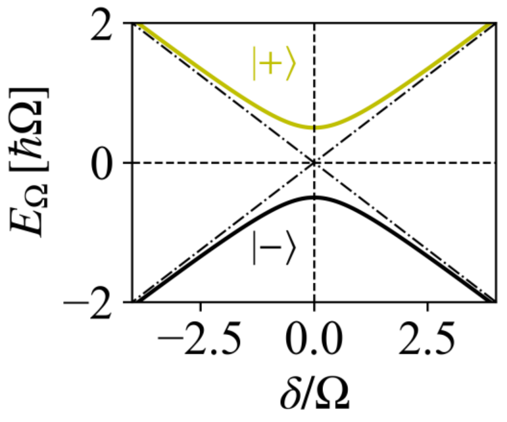


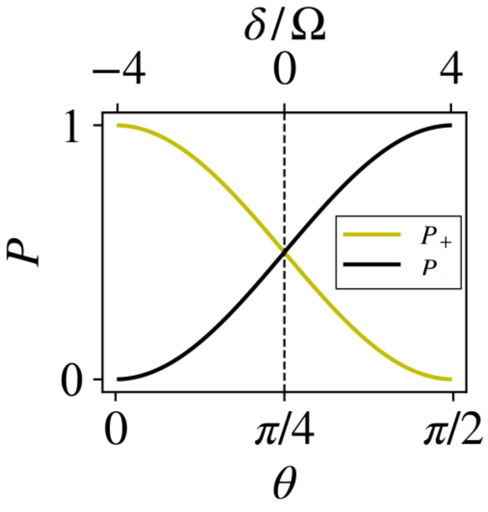
INTERACTIONS CONTROL IN ^{39}K SPIN MIXTURES

DRESSED STATE PICTURE:

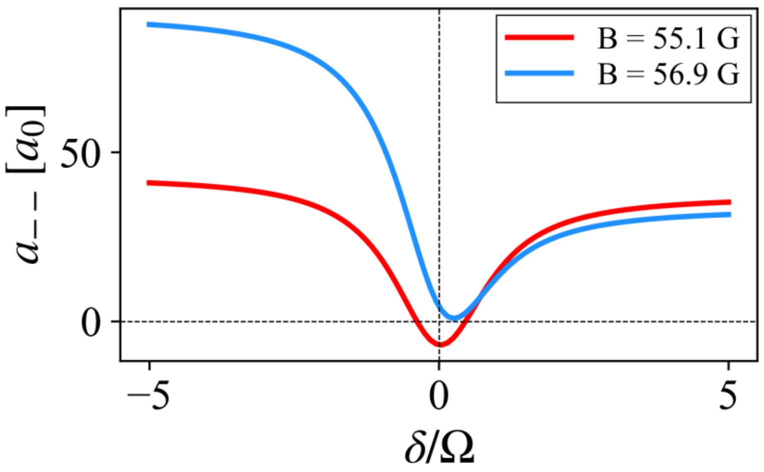


radio-frequency coupling





SCATTERING IN THE $|--\rangle$ CHANNEL: (low density regime)



$$\cot(2\theta) = \frac{\delta}{\Omega}$$

θ: spin-mixture polarization angle

$$P = 2\cos\theta^2 - 1$$

$$a_{--}^{(0)} = \sin^4\theta\, a_{11} + \cos^4\theta\, a_{22} + 2\sin^2\theta\cos^2\theta\, a_{12}$$

J. Sanz et al *Phys. Rev. Lett.* 128 (1 2022)

PERTURBATIVE REGIME

For higher densities the spin mixture composition adapts to minimize the energy:

$$\frac{\partial(E/N)}{\partial\theta} = 0 \longrightarrow \boxed{\cot(2\theta) = \frac{\delta}{\Omega} + 2\gamma\cos(2\theta) - \alpha/2}$$

$$\alpha = \frac{n(g_{\downarrow\downarrow} - g_{\uparrow\uparrow})}{\hbar\Omega}$$

$$\gamma = \frac{n\bar{g}}{\hbar\Omega}$$

$$\gamma \ll 1$$

TAYLOR EXPANSION AROUND n=0:

$$\frac{E(n)}{N} \approx \left.\frac{E(n)}{N}\right|_{n=0} + n\frac{d}{dn}\left.\frac{E(n)}{N}\right|_{n=0} + \frac{n^2}{2}\frac{^2}{dn^2}\left.\frac{E(n)}{N}\right|_{n=0} + \mathcal{O}(n^3)$$

$$\frac{E(n)}{N} \approx -E_\Omega + \frac{n}{2}g_2 + \frac{n^2}{2}g_3$$

effective 3-body energy correction

$g_3 \propto 1/\Omega$

A. Hammond et al. Phys. Rev. Lett (2022)
S. Tiengo et al. *Phys. Rev. A* 111 (5 2025)

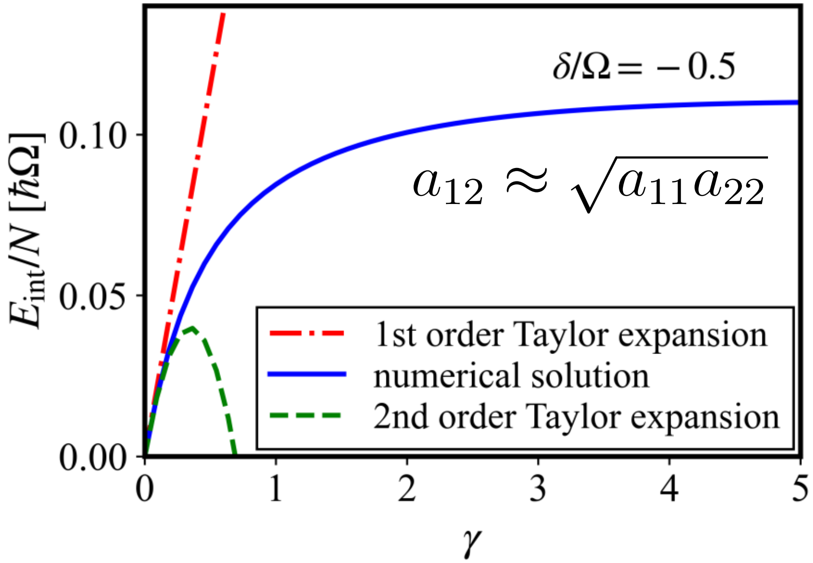
SRONGLY NON-LINEAR REGIME

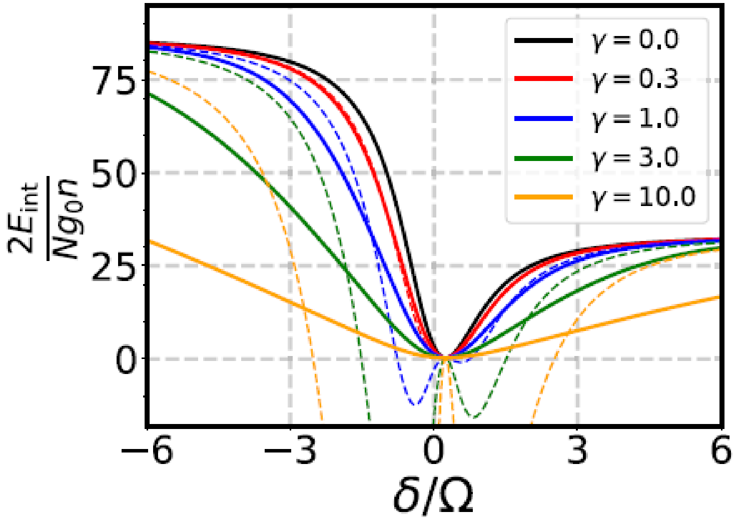
$$\cot(2\theta) = \frac{\delta}{\Omega} + 2\gamma\cos(2\theta) - \alpha/2$$

$$\alpha = \frac{n(g_{\downarrow\downarrow} - g_{\uparrow\uparrow})}{\hbar\Omega}$$

$$\gamma = \frac{n\bar{g}}{\hbar\Omega}$$

ENERGY SATURATION





- Highly non-linear regimes $(\gamma \gg 1)$ are experimentally accessible by tuning Ω towards low values.

- Magnetic filed stabilization up to a rms noise of $67\ \mu$G rms over 2 hours (over 57 G)

  S. Tiengo et al. *Rew. Sci. Instr.* 96(6) (2025)

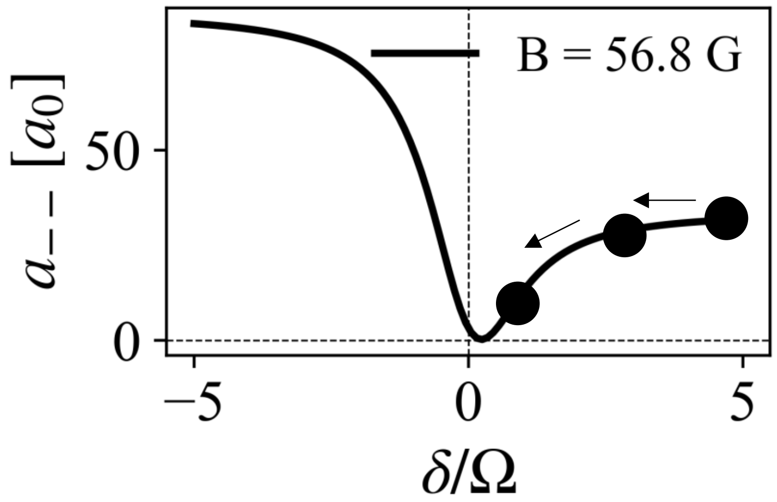
INTERACTION CONTROL FOR THE ^{39}K MIXTURE

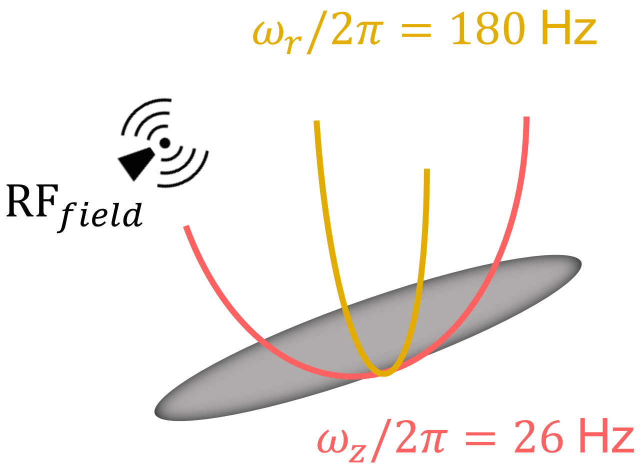
DRESSED CONDENSATE PREPARATION

1. Condensation in a cigar shaped trap (180×26) Hz
2. Set magnetic field B + B filed noise stabilization

3. Adiabatic Rapid Passage

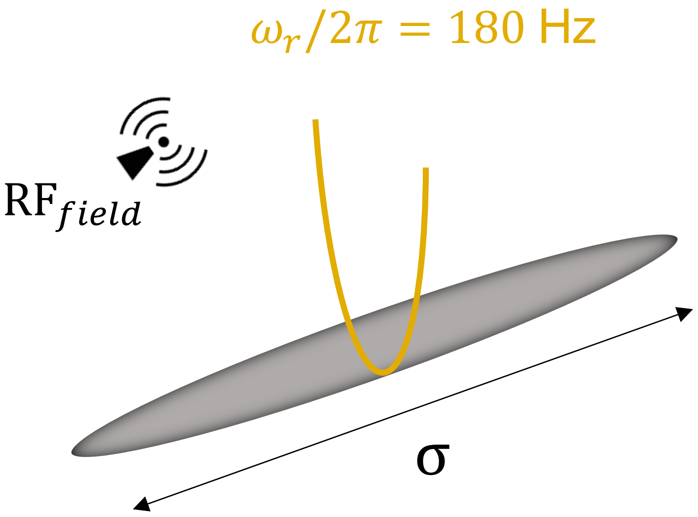
   for dressed condensate preparation

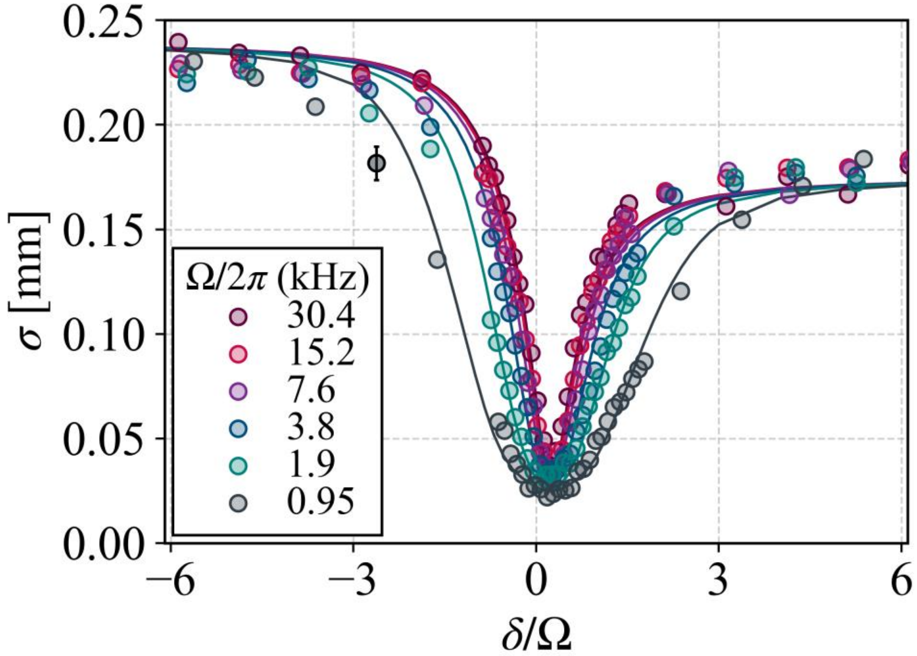




CONDENSATE 1D EXPANSION

1D expansion for $62.6\ ms$





Theory:

1. GP imaginary time for simulating dressed ground state initial solution
2. One-dimensional real time expansion dynamics

R. Eid et al. *Phys. Rev. A* 112 (12 2025)

CONCLUSION

- We demonstrated to have a precise control of interactions, from weakly to strongly nonlinear regimes

- Observation of energy saturation as a consequence of density-dependent spin-mixture composition

OUTLOOK

- Investigation of novel nonlinear effects in the physics of condensates
- Modulational instability study to characterize three-body correction

Thank you
for your attention

Thomas Bourdel

Sara Tiengo (3rd y PhD)
Maé Lévy (1st y PhD)
Nathan Honorè (1st y PhD)
Gabin Chateau (intern)
Roy Eid (former Phd)

DENSITY DEPENDENT MIXTURE POLARIZATION

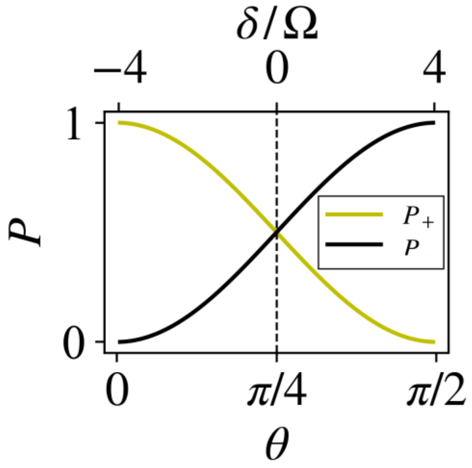
$$|\psi_1| = \sqrt{n}\cos\theta \qquad |\psi_2| = \sqrt{n}\sin\theta. \qquad \theta \in [0, \pi/2]$$

$$\frac{E}{N} = \frac{g_{11}}{2}|\psi_1|^4 + \frac{g_{22}}{2}|\psi_2|^4 + g_{12}|\psi_1|^2|\psi_2|^2 + \frac{\hbar\delta}{2}(|\psi_1|^2 - |\psi_2|^2) + \hbar\Omega(|\psi_1||\psi_2|) \qquad \gamma = \frac{n\bar{g}}{\hbar\Omega}$$

$\gamma = 0$

RABI REGIME

$$\cot(2\theta) = \frac{\delta}{\Omega}$$



$\gamma \neq 0$

NON-LINEAR RESPONSE REGIME

Density dependent spin mixture and energy

$$\frac{\partial(E/N)}{\partial\theta} = 0 \longrightarrow \cot(2\theta) = \frac{\delta}{\Omega} + 2\gamma\cos(2\theta) - \alpha/2$$

<u>Perturbative regime</u>

$\gamma \ll 1$

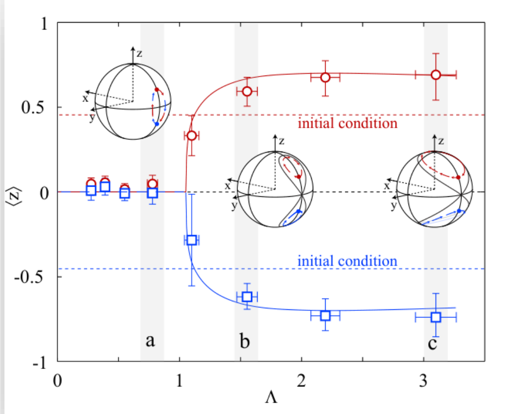
<u>Strongly non-linear regime</u>

$\gamma \geq 1$

RICH NON-LINEAR PHYSICS

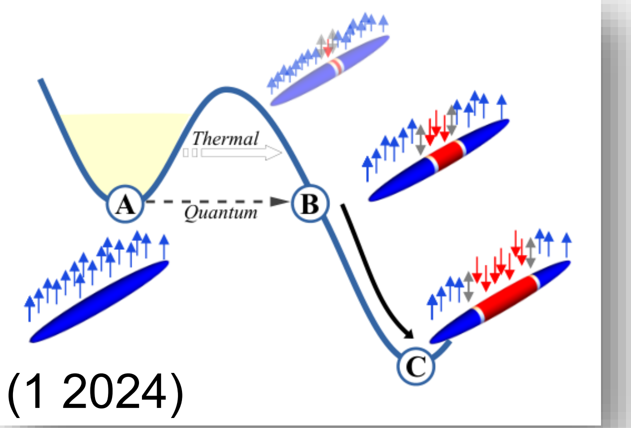
MAGNETIZATION
PITCHFORK BIFURCATION
with order parameter

$$\gamma = \frac{n\bar{g}}{\hbar\Omega}$$



T. Zibold et al. *Phys. Rev. Lett.* 105 (11 2010)

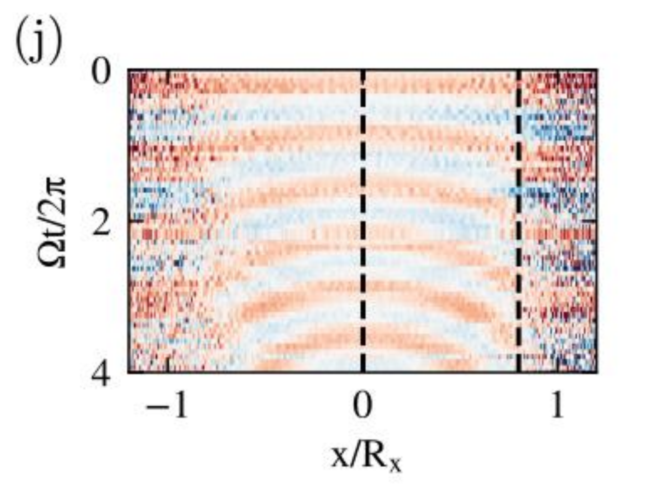
METASTABILITY
and
HYSTERESIS



A. Zanesini et al. *Nat. Phys.* (1 2024)

INTERNAL JOSEPHSON DYNAMICS

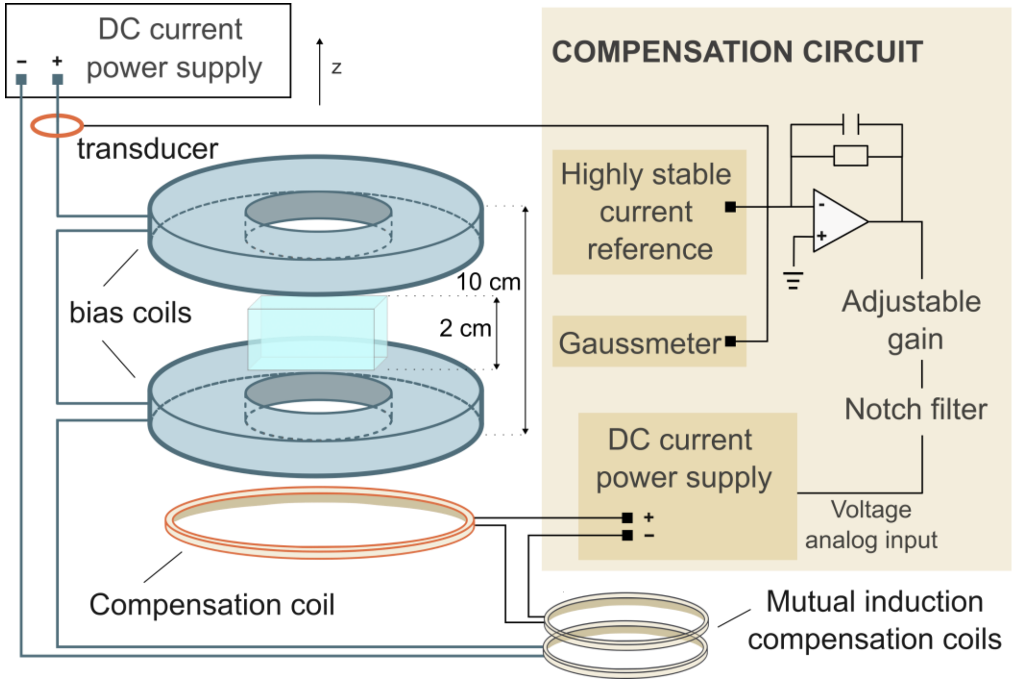
Density dependent
spin polarization and dynamics



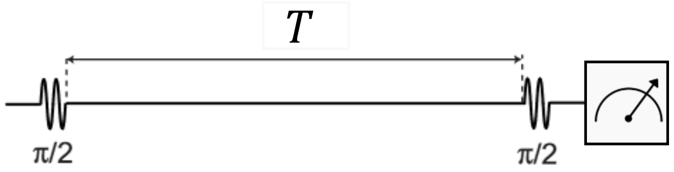
A. Farolfi et al. *Phys. Rev. A* (8 2021)

FEEDFORWARD CIRCUIT

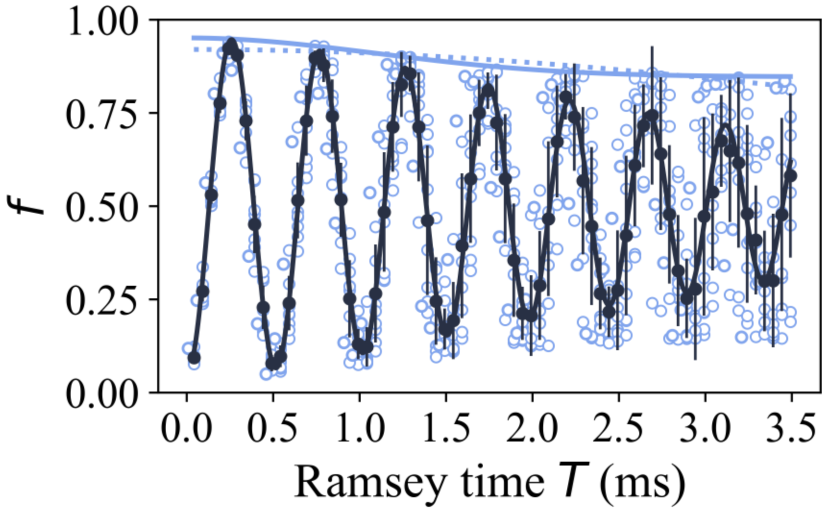
An additional compensation coil fed by a feedforward circuit for compensating
coil current fluctuations and **external magnetic field noise**



B = 57 G



$$\mathrm{mean}[f] = \frac{1}{2}\left(1 - C(T)\,\mathrm{e}^{-\frac{1}{2}(\sigma T)^2}\cos(\delta_0 T)\,T + \phi\right)$$



Long-term stability
(over 2 hours)
$rms = \mathbf{71(4)\ \mu G}$

1 ppm stability
(<100 Hz)

S. Tiengo, R. Eid, M. Apfel, G. Brulin, and T. Bourdel, "A simple magnetic field stabilization technique for atomic Bose-Einstein condensate experiments", *Review of Scientific Instruments, 96(6), (2025)*

PERTURBATIVE REGIME

$$\cot(2\theta) = \frac{\delta}{\Omega} + 2\gamma\cos(2\theta) - \alpha/2$$

$$\alpha = \frac{n(g_{\downarrow\downarrow} - g_{\uparrow\uparrow})}{\hbar\Omega}$$

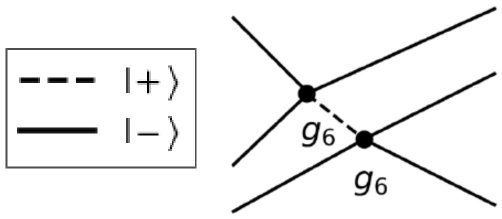
$$\gamma = \frac{n\bar{g}}{\hbar\Omega}$$

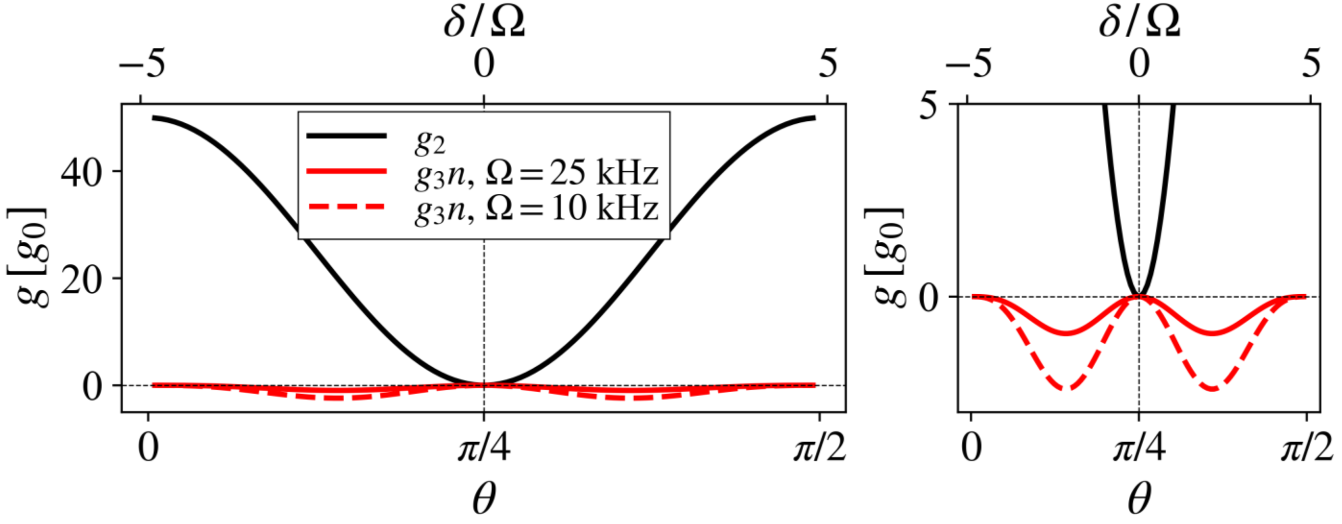
TAYLOR EXPANSION AROUND n=0: $\gamma \ll 1$

$$\frac{E(n)}{N} \approx \frac{E(n)}{N}\Big|_{n=0} + n\frac{d}{dn}\frac{E(n)}{N}\Big|_{n=0} + \frac{n^2}{2}\frac{{}^{2}}{dn^2}\frac{E(n)}{N}\Big|_{n=0} + \mathcal{O}(n^3)$$

$$\frac{E(n)}{N} \approx -E_\Omega + \frac{n}{2}g_2 + \frac{n^2}{2}g_3$$

$g_3 \propto 1/\Omega$

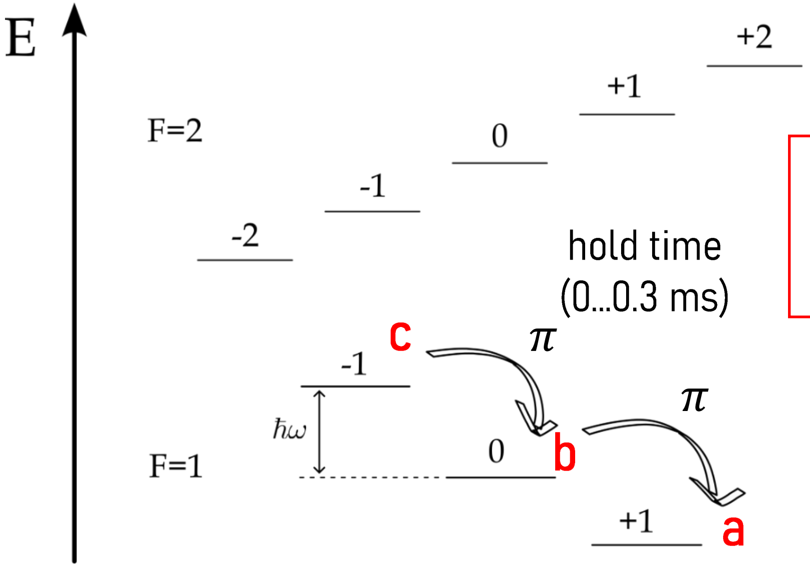




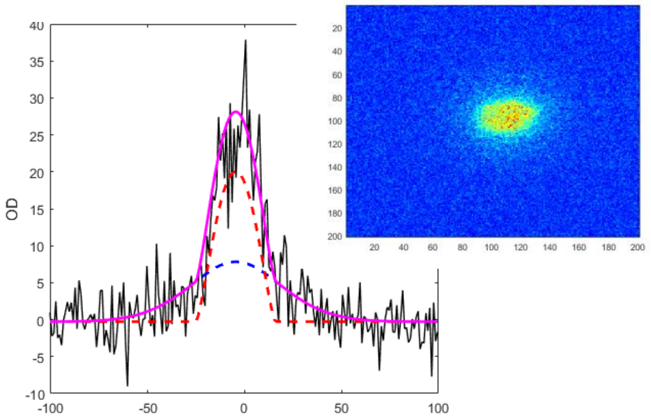
S. Tiengo et al. *Phys. Rev. A* 111 (5 2025)

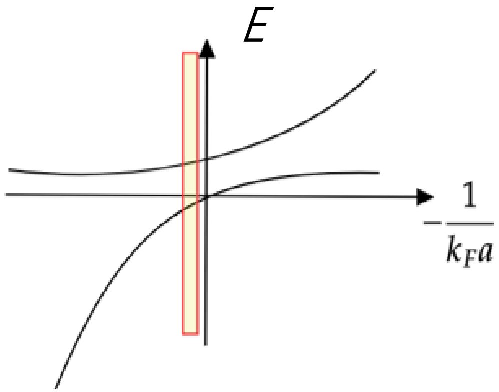
ONGOING WORK: quenches to resonance via RF pulses

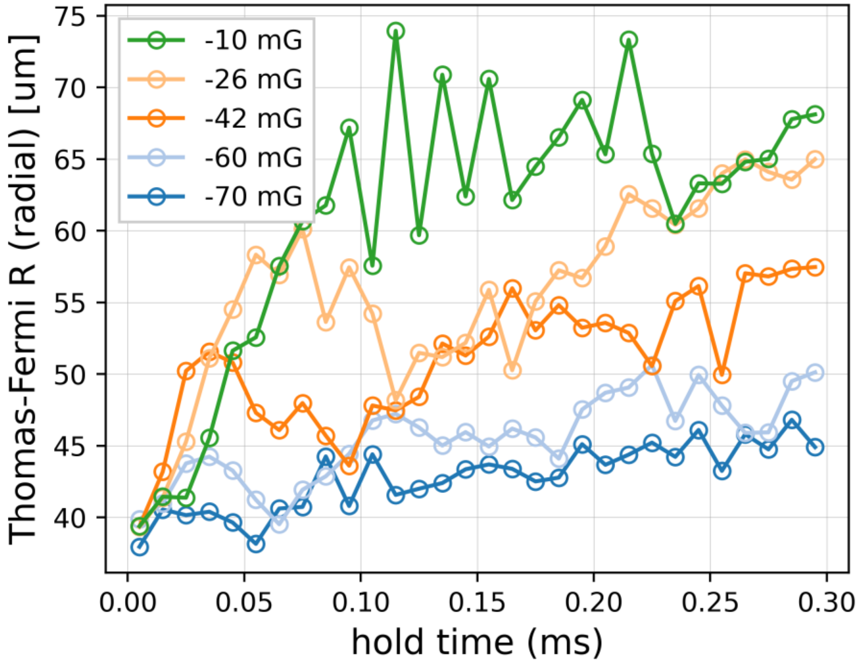
b-b resnonance at 58.9 G



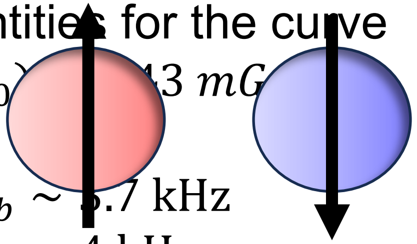
$a_{cc} \sim 30\ a_0$
a_{bb} resonant
$a_{bc} \sim -50\ a_0$







Few quantities for the curve at $(B - B_0)$ [illegible]3 mG

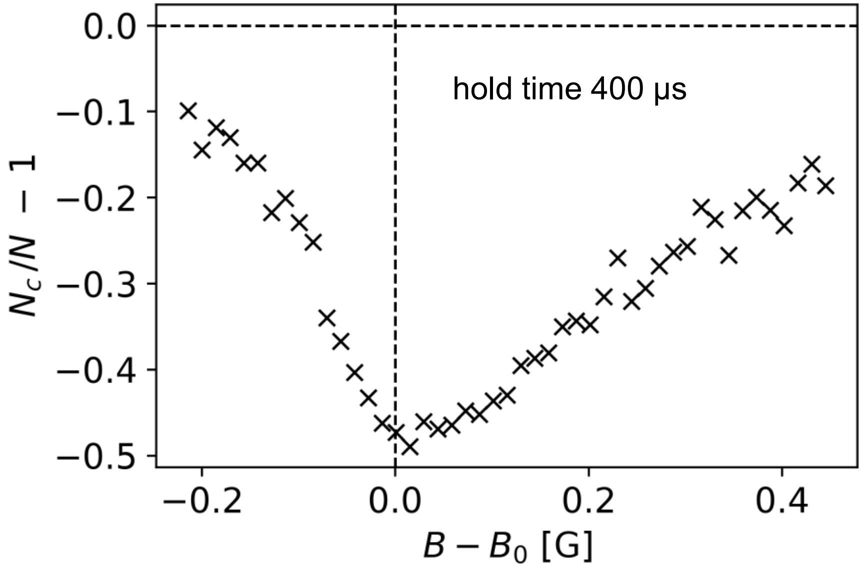


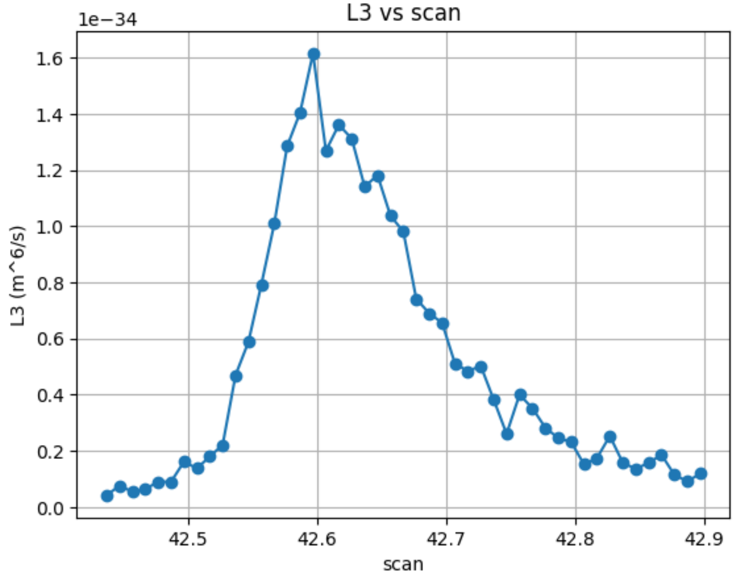
$E_b \sim$ [illegible].7 kHz
$E_F \sim 4$ kHz
$t_F \sim 0.23\ ms$
$k_F a \sim \sqrt{2}$

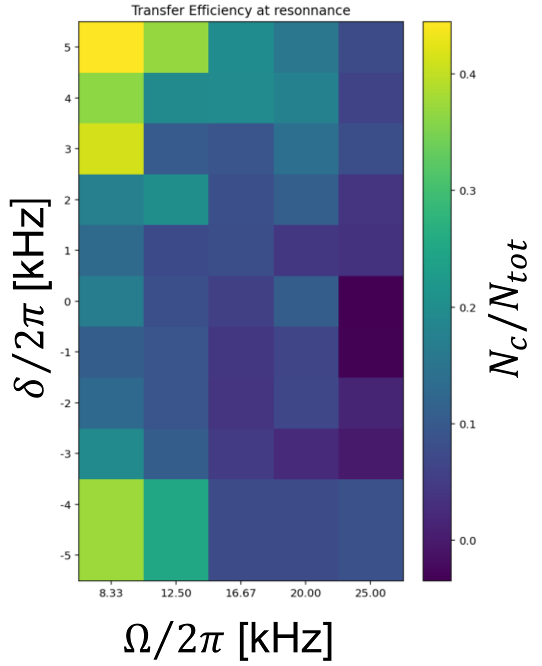
ONGOING WORK: quenches to resonance via RF pulses

Losses at T_c:

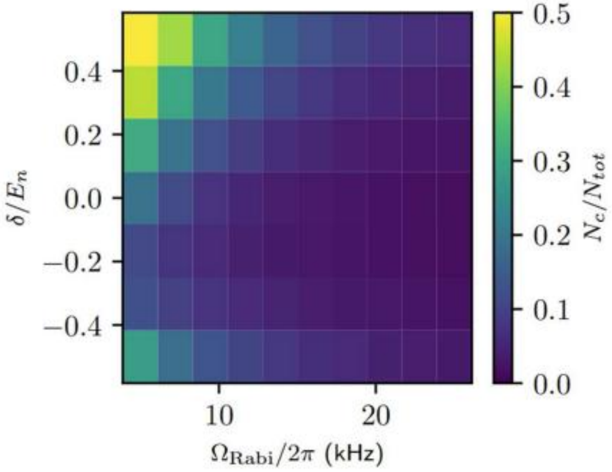
$$N = 8 \times 10^5$$
$$(\omega_r, \omega_z)/2\pi = (180, 40) Hz$$
$$T = 300\ nK$$
$$\lambda_{th} = 0.5\ \mu m$$
$$k = 12 \times 10^6\ m^{-1}$$



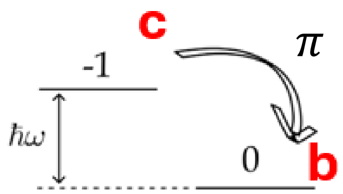




Theory from
S. Kokkelmans' team



RF transfer efficiency



$E_F \sim 10$ kHz

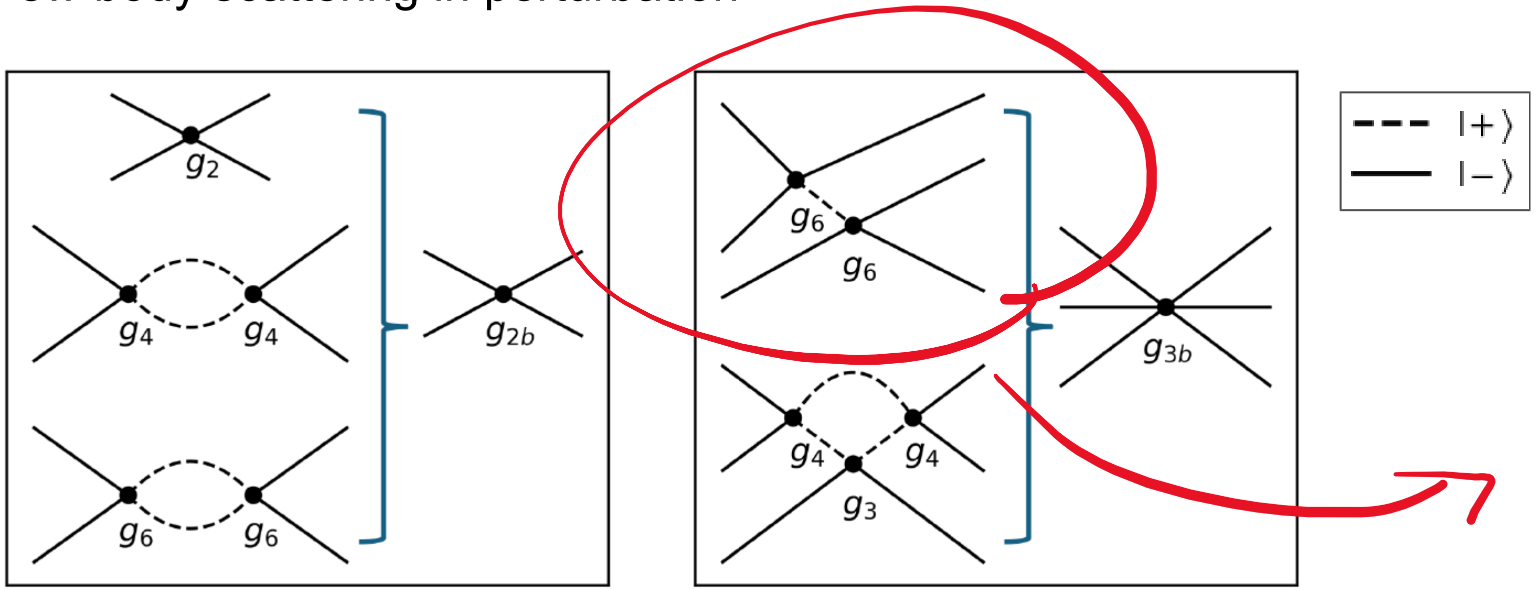
we look at atoms left in c

HIGHER ORDER INTERACTIONS: strong coupling

- Perturbative expansion of the condensate energy

$$E = E^{(0)} + \lambda E^{(1)} + \lambda^2 E^{(2)} + \lambda^3 E^{(3)} + \mathcal{O}(\lambda^4)$$

- Few-body scattering in perturbation



$$T_{i\to f} = \langle \psi_f^{2b} | T | \psi_i^{2b} \rangle$$

$$T_{i\to f} = \langle \psi_f^{3b} | T | \psi_i^{3b} \rangle$$

responsible for a 3-body

mean-field energy term

$g_{3b} \propto 1/\Omega$

- S. Tiengo, R. Eid, T. Bourdel, "Three-body interactions in Rabi-coupled Bose gases: a perturbative approach", *Phys. Rev. A* 111 (5 2025)
- A. Hammond, L. Lavoine, T. Bourdel, Tunable 3-body interactions in coherently driven two-component Bose-Einstein condensate, *Phys. Rev. Lett.* 128, 083401 (2022).
- L. Lavoine, A. Hammond, A. Recati, D.S. Petrov, T. Bourdel, Beyond-mean-field effects in Rabi-coupled two-component Bose-Einstein condensate, *Phys. Rev. Lett.* 127, 203402 (2021).