Cold Atoms, Spin Squeezing and Quantum Metrology

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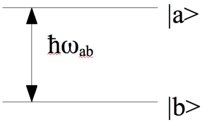
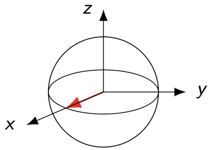
Plan

Atomic sensors : Projection noise and Squeezing

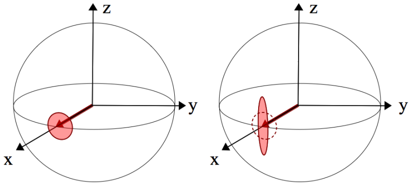
Two-level system



spin 1/2



N atoms : collective spin $N/2$



Projection noise : $s_y = \pm\frac{1}{2}$, $\Delta s_y = \frac{1}{2}$ $\qquad \Delta S_y = \frac{\sqrt{N}}{2}$ $\qquad \Delta S_y = \xi\frac{\sqrt{N}}{2}$

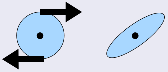
Angular uncertainty on the collective spin position $\Delta\varphi$

- **Coherent spin state for independent atoms :** $(\Delta\varphi)_{\mathrm{CSS}} = \frac{1}{\sqrt{N}}$
- **Spin-squeezed state $\xi < 1$ for correlated atoms :** $(\Delta\varphi)_{\mathrm{SSS}} = \frac{\xi}{\sqrt{N}}$

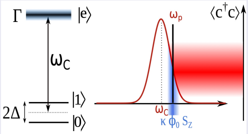
Different methods to realize spin squeezing

Non-linear evolution from atom-atom intercations

$$H_{NL} = \hbar \chi S_z^2$$

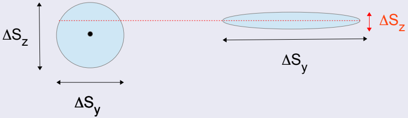


Cavity backaction : interaction with light in a cavity



$$H = \hbar(\omega_{at} + \kappa\phi_0 c^\dagger c) S_z \quad ; \phi_0 \propto \frac{g_0^2}{\Delta}$$

Quantum non demolition measurement



Experimental realisations of a spin-squeezed state

Non linear Hamiltonian $H_{NL} = \hbar\chi S_z^2$

- **Two components BEC :** $N \simeq 10^2 - 10^3$, $\xi^2 \simeq 0.15$

  Oberthaler, *Nonlinear atom interferometer surpasses classical precision limit* Nat. (2010).

  Treutlein, *Atom-chip-based generation of entanglement for quantum metrology* Nat. (2010).

- **Cavity backaction :** $N \simeq 10^2 - 10^4$, $\xi^2 \simeq 0.3$

  Vuletič, *Implementation of cavity squeezing of a collective atomic spin* PRL (2010).

  Vuletič, *Entanglement on an optical atomic-clock transition* Nat. (2020).

Spin squeezing by QND measurement

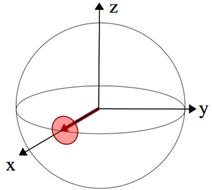
- **Large spin squeezing :** $N \simeq 10^5$, $\xi^2 \simeq 10^{-2}$

  Kasevich, *Measurement noise 100 times lower than the quantum-projection limit using entangled atoms* Nat. (2016).

- **Long-lived spin squeezing**

  Reichel, *Observing spin-squeezing under spin-exchange for a second*, PRX Quantum (2023).

Dynamical generation of spin squeezing in a BEC



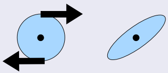
$$|S_x = N/2\rangle = \frac{1}{\sqrt{N!}} \left(\frac{a^\dagger + b^\dagger}{\sqrt{2}} \right)^N |0\rangle = \sum C_{N_a,N_b} |N_a, N_b\rangle$$

Interactions between cold atoms in a BEC close to $|S_x = N/2\rangle$

$$E(N_a, N_b) = \bar{E} + (\mu_a - \mu_b) S_z + \frac{\partial_{N_a}\mu_a + \partial_{N_b}\mu_b - \partial_{N_a}\mu_b - \partial_{N_b}\mu_a}{2} S_z^2 + \dots$$

Kerr-type Hamiltonian → spin-squeezing

$$H_{NL} = \hbar\chi S_z^2$$

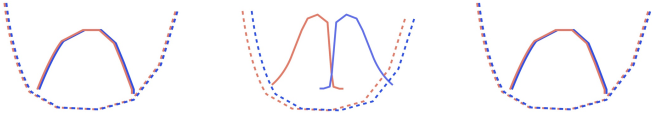


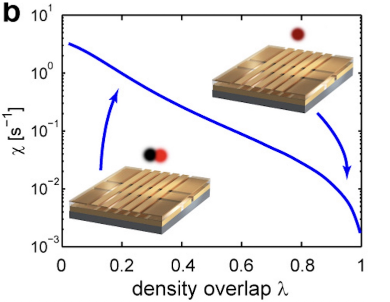
$$S_z = \frac{N_a - N_b}{2}$$

Dynamical control of the non-linearity

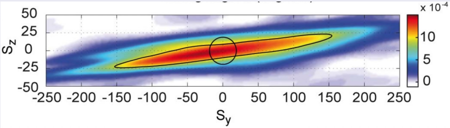
Changing the overlap between ϕ_a and ϕ_b (theory & experiment)

$\chi = 0$ $\qquad$ $\chi \neq 0$ $\qquad$ $\chi = 0$





Spin-squeezed 2-component BEC

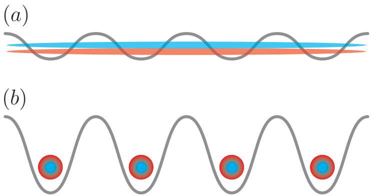


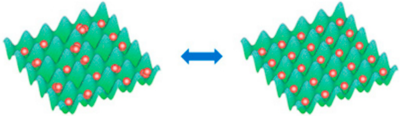
- In one spatial mode $C = 0.88$
- Single parameter estimation

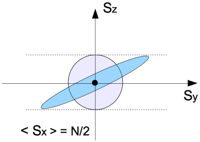
Riedel, P. Böhi, Y. Li, T. Hänsch, A. Sinatra, P. Treutlein, *Atom-chip-based generation of entanglement for quantum metrology* Nature (2010).

Squeezed-Mott state with a two component BEC

Adiabatically raise a lattice in a two component BEC (theory)







Squeezed-Mott state

- Compact arrangement + internal state coherence/correlations
- Spatially distributed entanglement for multiparameter estimation

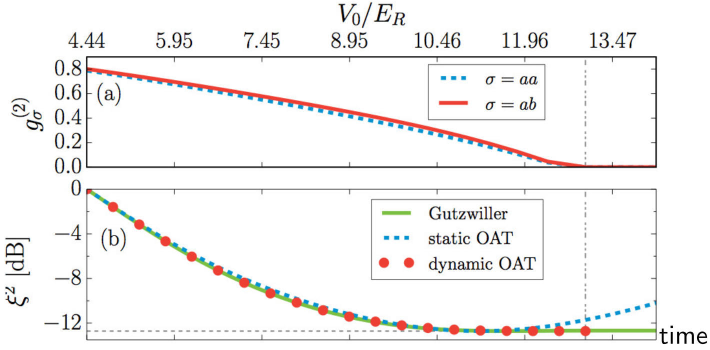
D. Kajtoch, E. Witkowska, A. Sinatra *Spin-squeezed atomic crystal*, EPL (2018)

Correlations through the Mott transition (3D)

On-site correlation functions

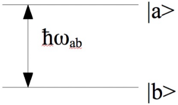
$$g^{(2)}_{aa}(t) = \frac{\langle a_i^\dagger a_i^\dagger a_i a_i \rangle}{\langle a_i^\dagger a_i \rangle^2} \qquad g^{(2)}_{ab}(t) = \frac{\langle a_i^\dagger b_i^\dagger a_i b_i \rangle}{\langle a_i^\dagger a_i \rangle \langle b_i^\dagger b_i \rangle}$$

Squeezing parameter $\xi^2(t)$ across the ramp $V_0(t) V_i + (V_f - V_i)\frac{t}{t_{\rm best}}$



Beyond spin squeezing with $H_{\mathrm{NL}} = \chi S_z^2$

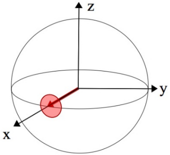
N two-level atoms



Collective spin

$$\hat{S}_x = \frac{a^\dagger b + b^\dagger a}{2},\ \hat{S}_y = \frac{a^\dagger b - b^\dagger a}{2i},$$

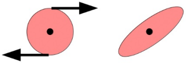
$$\hat{S}_z = \frac{a^\dagger a - b^\dagger b}{2}$$

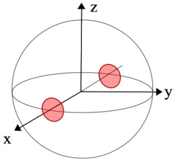


Initial state : Phase state

$$|\varphi\rangle \equiv \frac{1}{\sqrt{N!}}\left(\frac{e^{i\varphi/2}a^\dagger + e^{-i\varphi/2}b^\dagger}{\sqrt{2}}\right)^N |0\rangle$$

Evolution $H = \chi S_z^2$
Squeezed state





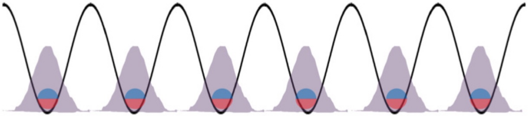
Schrödinger cat state

$$|\psi_{\mathrm{cat}}\rangle = \frac{|\varphi = 0\rangle + |\varphi = \pi\rangle}{\sqrt{2}}$$

[See also, in $S = 8$ dysprosium atoms *Quantum-enhanced sensing using non-classical spin states of a highly magnetic atom*, Nascimbene group, Nat. Comm. (2018)]

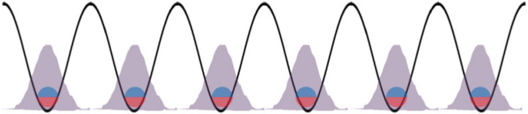
Exact numerical simulations for $N = 6$ in 1D

For $t_{\mathrm{ramp}} = t_{\mathrm{cat}}$, preparation of Greenberger-Horne-Zeilinger state



$$\frac{|a\rangle+|b\rangle}{\sqrt{2}} \quad \frac{|a\rangle+|b\rangle}{\sqrt{2}} \quad \frac{|a\rangle+|b\rangle}{\sqrt{2}} \quad \frac{|a\rangle+|b\rangle}{\sqrt{2}} \quad \frac{|a\rangle+|b\rangle}{\sqrt{2}} \quad \frac{|a\rangle+|b\rangle}{\sqrt{2}}$$

$$+$$



$$\frac{|a\rangle-|b\rangle}{\sqrt{2}} \quad \frac{|a\rangle-|b\rangle}{\sqrt{2}} \quad \frac{|a\rangle-|b\rangle}{\sqrt{2}} \quad \frac{|a\rangle-|b\rangle}{\sqrt{2}} \quad \frac{|a\rangle-|b\rangle}{\sqrt{2}} \quad \frac{|a\rangle-|b\rangle}{\sqrt{2}}$$

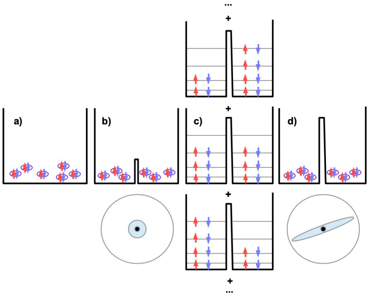
$$|\Psi\rangle_{\mathrm{GHZ},N=6} = \frac{e^{i\pi/4}}{\sqrt{2}} \left[|0\rangle_{MI} + i|\pi\rangle_{MI}\right]$$

M. Plodzien, M. Koscielski, E. Witkowska, A. Sinatra *Producing and storing spin-squeezed and Greenberger-Horne-Zeilinger states in a one-dimensional optical lattice* PRA (2020).

Remark : Large non-linearity with paired fermions ?

BEC in a double square well, spin-$\frac{1}{2}$ formed by external states

$$S_z = N_L - N_R \quad ; \quad H = \hbar\chi S_z \quad ; \quad \hbar\chi = \frac{d\mu}{dN}$$



$$\chi^{\mathrm{Bose}} = \frac{\rho g}{N} \quad \text{with} \quad g = \frac{4\pi\hbar^2}{m} a$$

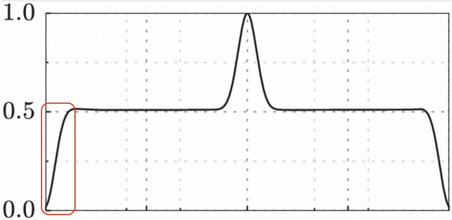
$$\chi^{\mathrm{Fermi}} = \frac{2}{3}\frac{\epsilon_F}{N} \quad \text{with} \quad \epsilon_F \propto \rho^{2/3}$$

$$a \to 0^- \qquad \mu \to \epsilon_F$$

$$\boxed{\frac{\chi^{\mathrm{Fermi}}}{\chi^{\mathrm{Bose}}} \simeq \frac{\epsilon_F}{\rho g} \simeq \frac{1}{(\rho a^3)^{1/3}} \gg 1}$$

A. Sinatra *Spin squeezed states for metrology* APL (2022)

Scaling lows for squeezed and over-squeezed states



- **Evolution with** $H = \hbar\chi S_z^2$
- **Fisher information** $\mathcal{F}/N^2$ **as a function of time** χt
- **Limits the squeezing** $\xi^2 \geq \frac{N}{\mathcal{F}}$

For over-squeezed states : MAI (measurement after interaction)

$|\psi\rangle = e^{i\chi t S_z^2} e^{i\theta S_{\vec{n}}} e^{-i\chi t S_z^2} |Ox\rangle^{\otimes N}$ **measure** $S_{\vec{m}}$ M. Schleier-Smith, PRL (2016)

For squeezed and over-squeezed states $\frac{1}{N} < \chi t < \frac{1}{\sqrt{N}}$, the quantum gain ξ^{-2} has the same N-scaling as $\mathcal{F}$

.

For $\chi t \propto N^{-\alpha}$ **with** $\alpha \in]1/2, 1]$ **one has :** $\xi^2 \propto N^{2(\alpha-1)}$

Y. Baamara, A. Sinatra, M. Gessner, *Squeezing of nonlinear spin observables by one axis twisting in the presence of decoherence: An analytical study*, Comptes Rendus Physique 23, 1-26 (2022).

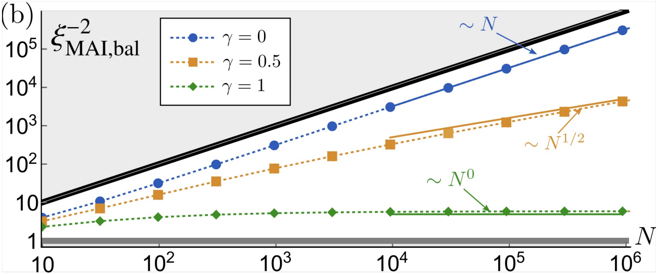
Decoherence changes the scalings laws

Balistic dephasing model

$$H_{\mathrm{bal}} = \hbar\chi\left(S_z^2 + DS_z\right) \quad \text{with} \quad \langle D^2\rangle = \epsilon N^\gamma$$

Limitation of the maximal quantum gain in the limit of large N

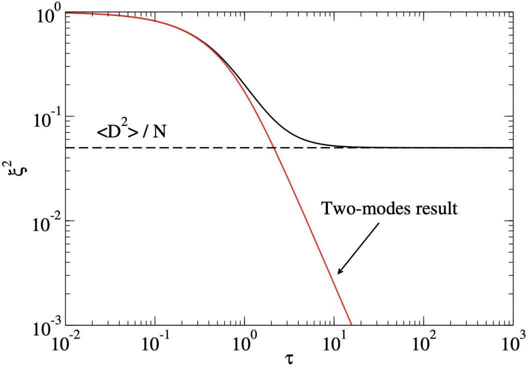
$$\xi^{-2} \leq \frac{N^{1-\gamma}}{4\epsilon} \quad \Leftrightarrow \quad \xi^2 \geq 4\frac{\langle D^2\rangle}{N}$$



Y. Baamara, A. Sinatra, M. Gessner, *Scaling laws for the sensitivity enhancement of non-Gaussian spin states*, PRL (2021).

BEC phase dynamics at finite temperature

$$\theta_a(t) - \theta_b(t) \underset{\text{large } t}{\simeq} -\overbrace{\frac{\partial_{\langle N_a \rangle} \mu_\Phi}{\hbar} t}^{\chi t} \left[N_a - N_b + \overbrace{\sum_k \partial_{\mu_\Phi} \epsilon_k (n_{ka} - n_{kb})}^{D}\right]$$

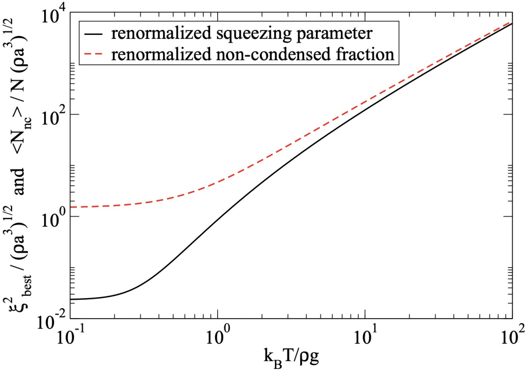


A. Sinatra, E. Witkowska, J.-C. Dornstetter, Y. Li, Y. Castin, *Limit of Spin Squeezing in Finite-Temperature Bose-Einstein Condensates*, PRL (2011), EPJ ST (2012).

Two component BEC : non-condensed fraction

Multimodal Bogoliubov description, thermodynamic limit

Spin squeezing limit for $N \to \infty$ $$\xi^2_{\min} \overset{K_B T > \mu}{\simeq} \left(\frac{N_{\rm nc}}{N} \right)$$



$k_B T \simeq \mu \simeq 10\hbar\omega$ **a gain by 30 on $\Delta\phi/(\Delta\phi)_{\rm NC}$ should be possible.**

Squeezing in two component BEC : effet of losses

Master equation description : 1, 2, 3-body losses

- **Spin-squeezing limit**

$$\boxed{\xi^2(t) \simeq \xi_0^2(t) + \frac{1}{3}\frac{N_{\text{lost}}(t)}{N}}$$

- **Evolution of spin-squeezed state in presence of losses**

$$\xi^2(t) - 1 = \left[\xi^2(0) - 1\right] e^{-\gamma t} .$$

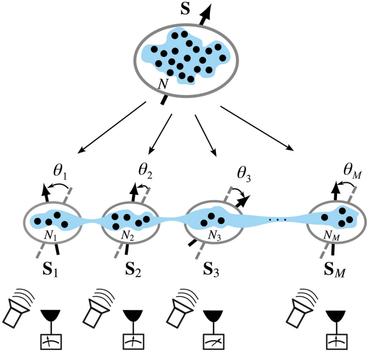
Y. Li, Y. Castin, and A. Sinatra, *Optimum Spin Squeezing in Bose-Einstein Condensates with Particle Losses*, PRL (2008).

- **Observation of a spin squeezed state over $1s$ (TACC exp)**

M.-Z. Huang, J. A. de la Paz, T. Mazzoni, K. Ott, A. Sinatra, C. L. G. Alzar, J. Reichel, *Self-amplifying spin measurement in a long-lived spin-squeezed state*, PRX Quantum (2023).

Multiparameter measurement with quantum gain

Spin-squeezed BEC as a source to measure a field in M points



- **The state is adapted to measure $\theta_1 + \theta_2 + \ldots + \theta_M$**
- **Local pulses π to reconfigure quantum correlations and measure other combinations of $\theta_1, \ldots, \theta_M$**

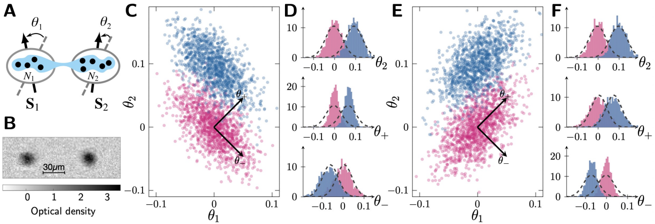
Yifan Li, L. Joosten, Y. Baamara, P. Colciaghi, A. Sinatra, P. Treutlein, T. Zibold *Multiparameter estimation with an array of entangled atomic sensors* Science (2026)

Example with two unknown phases θ_1 et θ_2

- θ_1, θ_2 small rotations of S_1, S_2 around axis Oy, estimated by :

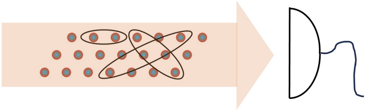
$$\theta_1 = \frac{S_1^z}{\langle S_1^x \rangle} \quad ; \quad \theta_2 = \frac{S_2^z}{\langle S_2^x \rangle}$$

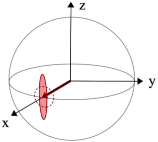
Experiments : $\theta_1 = 0$ et $\theta_2 = 0$ (pink) or $\theta_1 = 0$ et $\theta_2 \neq 0$ (blue).



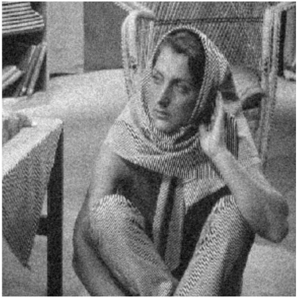
- Initial state : $(\theta_1 + \theta_2) \propto S_z$ estimated with quantum gain (left)
- Pulse changing the sign of S_2^z : $(\theta_1 - \theta_2) \propto (S_1^z - S_2^z)$ estimated with quantum gain (right)
- **With $-6{,}5$dB in the BEC $\rightarrow$ 33% reduction (standard dev) of quantum noise in the deux parameter estimateurs.**

Field sensing with single atoms (theory)





Left : independent atoms
Right : $-21{,}6$ dB, $H = \hbar\chi S_z^2$ plus decoherence, $N = 512 \times 512$



Compressed sensing : only $\mathcal{L}_{\mathcal{H}} < N$ Hadamard coefficients are measured

$N = 512 \times 512$

CSS, $\mathcal{L}_{\mathcal{H}} = 64^2$

SSS, $\mathcal{L}_{\mathcal{H}} = 64^2$

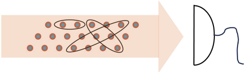
SSSD

Y. Baamara, M. Gessner, A. Sinatra, *Quantum-enhanced multiparameter estimation and compressed sensing of a field*, SciPost Physics, (2023)

Quantum noise reduction

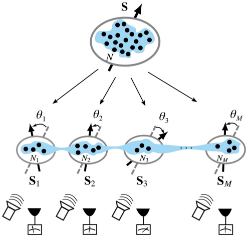
μ preparations of squeezed state ξ^2 with N atoms divided in M sites

- **Collective measurement**



$$\frac{\Delta\theta_k}{(\Delta\theta_k)_{CSS}} = \xi \quad \forall k$$

- **Local measurements**



$$\frac{\Delta\theta_k}{(\Delta\theta_k)_{CSS}} = \sqrt{\frac{M\xi^2}{1+(M-1)\xi^2}} \quad \forall k$$

$$\mathrm{Var}\,\theta_k = \frac{1}{\frac{1}{\mathrm{Var}\,\check{\theta}_{\mathrm{sq}}} + (M-1)\frac{1}{\mathrm{Var}\,\check{\theta}_{\mathrm{unsq}}}} = \bar{\sigma}^H$$

harm. ave. of eigenv. of initial Σ

In the estimation of some parameter combinations

- **ξ^2 is the noise reduction when estimating a single combination**
- **$\mathcal{L}_{\mathcal{H}}\xi^2$ is the noise reduction for $\mathcal{L}_{\mathcal{H}}$ combinations**
- **Compressed sensing, Spatial Fourier, Correlation functions ...**
  $\mathcal{L}_{\mathcal{H}} \ll M \ll N$

Conclusion

- Quantum projection noise in atomic sensors can be reduced with spin squeezing.
- Squeezing can be achieved in two-component BEC using atom-atom interactions.
- Atomic non-linearity can be controlled and both single-mode and multi-mode spin squeezed states can be prepared.
- Multi-mode squeezing enables Multiparameter estimation and field sensing

  $\Rightarrow$ Measuring spatial moments or Hadamard components

  $\Rightarrow$ Field mapping with compressed sensing