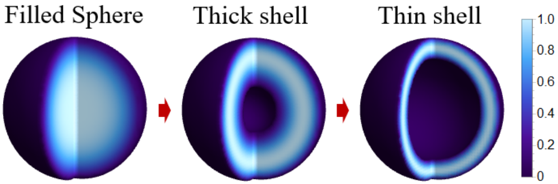
12.06.2026 / Frontiers in ultracold quantum gases

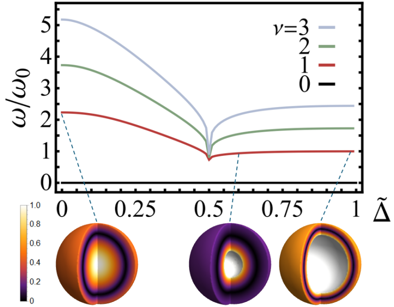
Dynamics of finite-temperature shell-shaped BECs

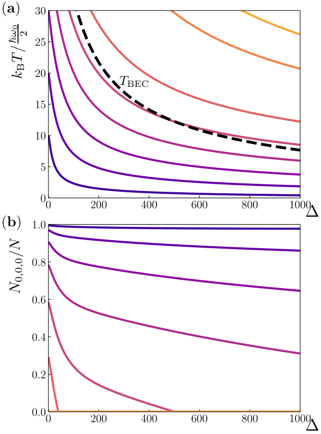
Brendan Rhyno

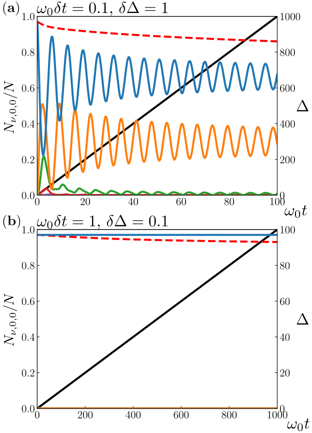
Theory of Quantum Sensing
(Lead: Dr. Naceur Gaaloul)
Institute of Quantum Optics
Leibniz University Hannover

Shell-shaped Bose-Einstein condensates



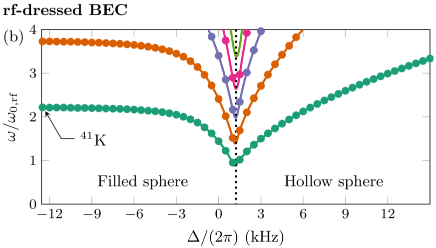






AVS Quantum Sci. 8, 010501 (2026)

Interesting properties of spherical condensates

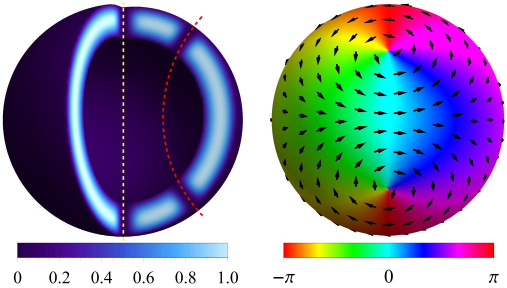


1

Collective excitation spectra show clear signatures of topological transition from filled to hollow sphere. [2]

In the case of a 2-sphere, topology enforces zero vortex circulation:

$$\sum_i n_i = 0$$



3

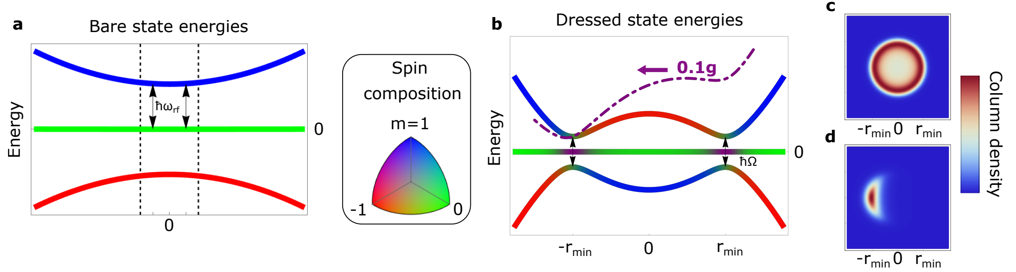
---

[1] Wolf et al., "Shell-shaped Bose-Einstein condensates based on dual-species mixtures", 2022.

[2] Padavić et al., "Physics of hollow Bose-Einstein condensates", 2017.

[3] Padavić et al., "Vortex-antivortex physics in shell-shaped Bose-Einstein condensates", 2020.

Shell-shaped condensates with rf-dressed potentials



[4]

Atomic bubbles can be created by engineering “dressed” (or “adiabatic”) shell-like confining potentials:[5]

❶ Trap atoms in a static magnetic field
❷ Apply radio frequency magnetic field
❸ (Rotating-frame transformation and rotating-wave approximation)

= creation of “dressed” potentials

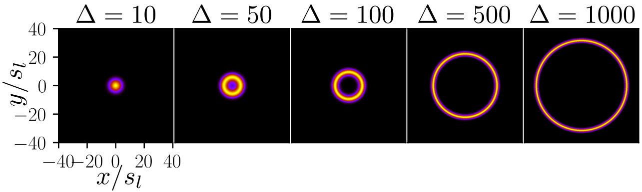
---

[4] Carollo et al., “Observation of ultracold atomic bubbles in orbital microgravity”, 2022.

[5] Zobay and Garraway, “Two-Dimensional Atom Trapping in Field-Induced Adiabatic Potentials”, 2001.

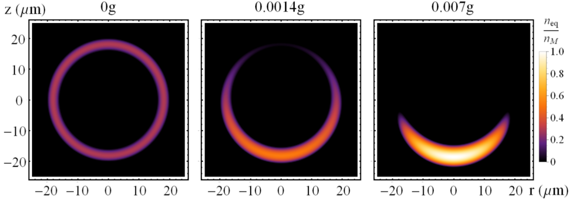
Shell-shaped condensates with rf-dressed potentials

Should be possible to inflate BEC into bubble adiabatically!



[6]

Problem: Earth's gravitational field leads to sag



[7]

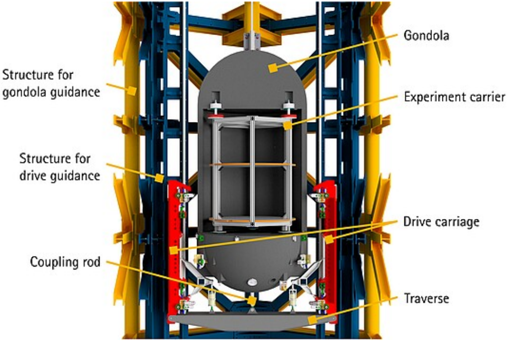
---

[6] Rhyno et al., "Thermodynamics in expanding shell-shaped Bose-Einstein condensates", 2021.

[7] Sun et al., "Static and dynamic properties of shell-shaped condensates", 2018.

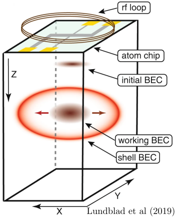
Einstein Elevator (EE)

- The Einstein Elevator (EE) drop tower at Leibniz University Hannover offers Terrestrial route for consistent microgravity conditions
- QUANTUMANIA project aims to create quantum bubbles in the EE

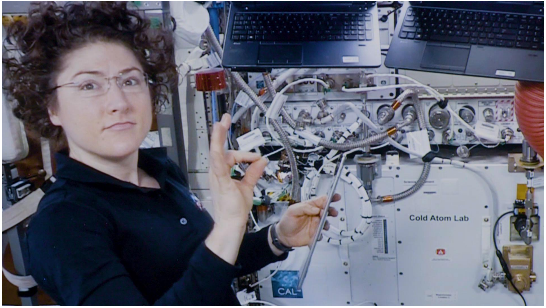


Cold Atom Lab (CAL)

- NASA's Cold Atom Lab (CAL) operates in perpetual free-fall aboard the International Space Station
- First observation of ultracold atomic bubbles

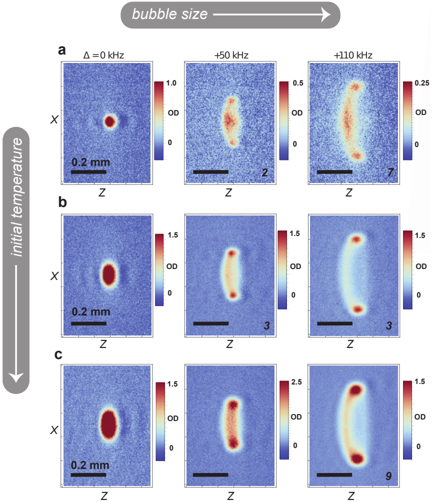


Lundblad et al (2019)

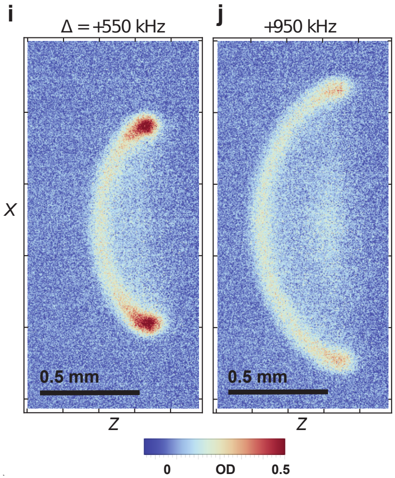


Observation of ultracold bubbles using CAL

- Initial temperatures: $T_i \approx 100\text{nK}$



- Extreme inflation to mm-scale sizes ($T_i \approx 1\mu\text{K}$) (not fully covered)



[8]

---

[8] Carollo et al., "Observation of ultracold atomic bubbles in orbital microgravity", 2022.

Thermodynamics in expanding shell-shaped condensates

Model: Weakly interacting Bose gas in a confining potential

$$\hat{H} = \int_{\mathbb{R}^3} d^3x \left[\hat{\psi}^\dagger(\vec{x}) \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{x})\right) \hat{\psi}(\vec{x}) + \frac{g}{2}\hat{\psi}^\dagger(\vec{x})\hat{\psi}^\dagger(\vec{x})\hat{\psi}(\vec{x})\hat{\psi}(\vec{x})\right]$$

where $g \geq 0$ represents a coarse-grained contact interaction.

Goal: Assuming thermal and diffusive equilibrium, compute expectation values using

$$\langle\hat{\mathcal{O}}\rangle = \frac{\mathrm{Tr}\left(e^{-\beta(\hat{H}-\mu\hat{N})}\,\hat{\mathcal{O}}\right)}{\mathrm{Tr}\left(e^{-\beta(\hat{H}-\mu\hat{N})}\right)} \quad , \quad \beta = \frac{1}{k_{\mathrm{B}}T}$$

Dilute case: $g = 0$ thermodynamics follow from Schrödinger Eq.

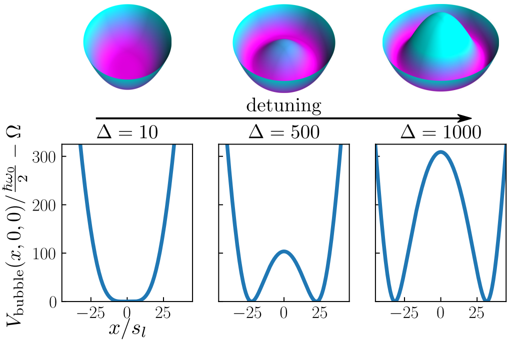
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{x})\right)\phi_\alpha(\vec{x}) = \varepsilon_\alpha\phi_\alpha(\vec{x}) \quad , \quad \varepsilon_0 < \varepsilon_1 \leq \varepsilon_2 \leq \dots$$

Idealized “bubble” trap

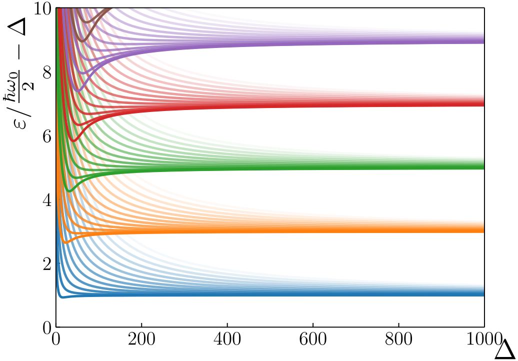
Capture salient features of experimental traps with isotropic potential:

$$V_{\text{bubble}}(\vec{x}) \propto \sqrt{\left[|\vec{x}|^2 - \Delta\right]^2 + (2\Omega)^2}$$

Bubble trap inflation



One-body spectrum ($\Omega = \Delta$)



Bubble trap thermometry – noninteracting ($g = 0$)

- BEC critical temperature:

$$N = \sum_{\alpha \neq 0} \frac{1}{e^{(\varepsilon_\alpha - \varepsilon_0)/k_\mathrm{B} T_\mathrm{BEC}} - 1}$$

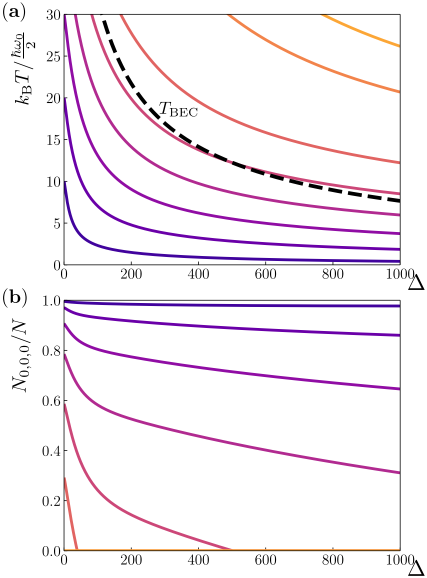
- Temperature as system inflates into a bubble adiabatically:

$$N(T, \mu) = \sum_\alpha f_\alpha$$

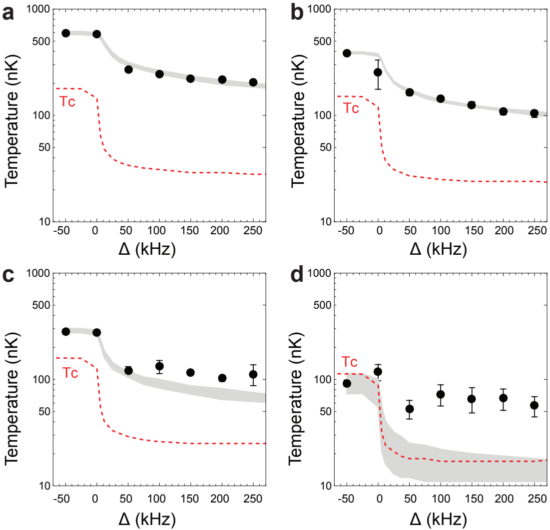
$$S(T, \mu) = k_\mathrm{B} \sum_\alpha [(1 + f_\alpha) \ln(1 + f_\alpha) - f_\alpha \ln f_\alpha]$$

where $f_\alpha = 1/(e^{\beta(\varepsilon_\alpha - \mu)} - 1)$.

- **Condensate depletion**: T_c decreases faster than temperature drops during adiabatic expansion!



CAL trap thermometry – noninteracting ($g = 0$)



- BEC critical temperature:

$$N = \frac{1}{\lambda_{T_c}^3} \int_{\mathbb{R}^3} d^3x \, \mathrm{Li}_{\frac{3}{2}}\left(e^{-V(\vec{x})/k_B T_c}\right)$$

- Temperature as system inflates into a bubble adiabatically:

$$N(T,\mu) = \frac{1}{\lambda_T^3} \int_{\mathbb{R}^3} d^3x \, \mathrm{Li}_{\frac{3}{2}}\left(e^{-\beta(V(\vec{x})-\mu)}\right)$$

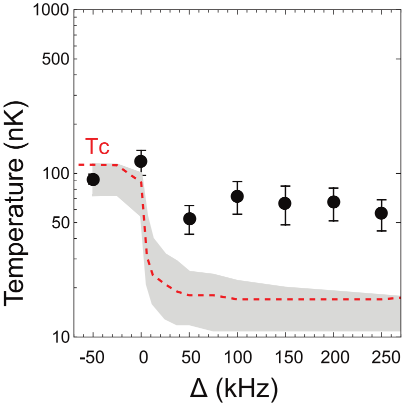
$$S(T,\mu) = \frac{k_B}{\lambda_T^3} \int_{\mathbb{R}^3} d^3x \left[\frac{5}{2} \mathrm{Li}_{\frac{5}{2}}\left(e^{-\beta(V(\vec{x})-\mu)}\right) + \beta(V(\vec{x})-\mu) \, \mathrm{Li}_{\frac{3}{2}}\left(e^{-\beta(V(\vec{x})-\mu)}\right) \right]$$

where $\lambda_T \equiv (2\pi\hbar^2/mk_B T)^{1/2}$ and $\mathrm{Li}_s(z) = \sum_{n=1}^{\infty} z^n/n^s$.

Kibble-Zurek mechanism

Correlation length and relaxation time **diverge** when a system is driven across a continuous phase transition using control parameter $\Delta(t)$:

$$\xi \sim |\Delta - \Delta_c|^{-\nu} \quad , \quad \tau \sim |\Delta - \Delta_c|^{-\nu z}$$

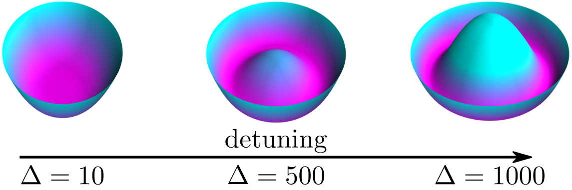


Breakdown of adiabaticity?

Nonequilibrium dynamics

Allow trap to vary dynamically in time:

$$V_{\text{bubble}}(t,\vec{x}) \propto \sqrt{\left[|\vec{x}|^2 - \Delta(t)\right]^2 + (2\Omega(t))^2}$$



Initial thermal state:

$$\hat{\rho}(t_0) = \frac{1}{\mathcal{Z}} e^{-\beta(\hat{H}(t_0)-\mu\hat{N})}$$

Time evolution:

$$\hat{\rho}(t) = \hat{U}(t,t_0)\,\hat{\rho}(t_0)\,\hat{U}^\dagger(t,t_0) \quad , \quad \hat{U}(t,t_0) = \mathcal{T}\exp\left(-\frac{i}{\hbar}\int_{t_0}^{t} dt'\,\hat{H}(t')\right)$$

Nonequilibrium dynamics (dilute gas)

Hamiltonian:

$$\hat{H}(t) = \int_{\mathbb{R}^3} d^3x \, \hat{\psi}^\dagger(\vec{x}) \left(-\frac{\hbar^2}{2m} \nabla^2 + V_{\text{bubble}}(t, \vec{x}) \right) \hat{\psi}(\vec{x})$$

Instantaneous diagonalization:

$$\hat{H}(t) = \sum_\alpha \varepsilon_\alpha(t) \, \hat{b}_\alpha^{S\dagger}(t) \hat{b}_\alpha^S(t) \quad , \quad \hat{b}_\alpha^S(t) \equiv \int_{\mathbb{R}^3} d^3x \, \phi_\alpha^*(t, \vec{x}) \, \hat{\psi}(\vec{x})$$

Goal: compute *instantaneous* single-particle energy eigenstate mode occupations

$$\begin{aligned} N_\alpha(t) &= \mathrm{Tr}\left(\hat{\rho}(t) \, \hat{b}_\alpha^{S\dagger}(t) \hat{b}_\alpha^S(t) \right) \\ &= \mathrm{Tr}\left(\hat{\rho}(t_0) \, \hat{b}_\alpha^{H\dagger}(t) \hat{b}_\alpha^H(t) \right) \quad , \quad \hat{b}_\alpha^H(t) \equiv \hat{U}^\dagger(t, t_0) \, \hat{b}_\alpha^S(t) \, \hat{U}(t, t_0) \end{aligned}$$

Nonequilibrium dynamics (dilute gas)

To facilitate calculation, work with *initial* single-particle energy eigenstate mode operators:

$$\hat{b}^{H}_{\alpha}(t,t_0) \equiv \hat{U}^{\dagger}(t,t_0)\,\hat{b}^{S}_{\alpha}(t_0)\,\hat{U}(t,t_0)$$

Heisenberg EOM:

$$i\hbar\partial_t \hat{b}^{H}_{\alpha}(t,t_0) = \left[\hat{b}^{H}_{\alpha}(t,t_0), \hat{H}^{H}(t)\right] = \cdots = \sum_{\alpha'} h_{\alpha\alpha'}(t,t_0)\,\hat{b}^{H}_{\alpha'}(t,t_0)$$

where

$$h_{\alpha\alpha'}(t,t_0) = \langle \alpha, t_0 | h(t) | \alpha', t_0 \rangle = \int_{\mathbb{R}^3} d^3x\, \phi^{*}_{\alpha}(t_0,\vec{x}) \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{bubble}}(t,\vec{x}) \right) \phi_{\alpha'}(t_0,\vec{x})$$

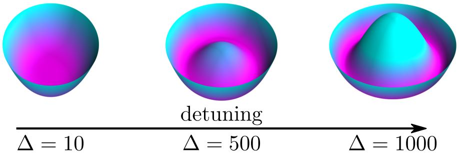
Formal solution:

$$\hat{b}^{H}_{\alpha}(t,t_0) = \sum_{\alpha'} \mathcal{U}_{\alpha\alpha'}(t,t_0)\,\hat{b}^{S}_{\alpha'}(t_0) \quad , \quad \mathcal{U}_{\alpha\alpha'}(t,t_0) = \langle \alpha, t_0 | \mathcal{T} \exp\left(-\frac{i}{\hbar} \int_{t_0}^{t} dt'\, h(t') \right) | \alpha', t_0 \rangle$$

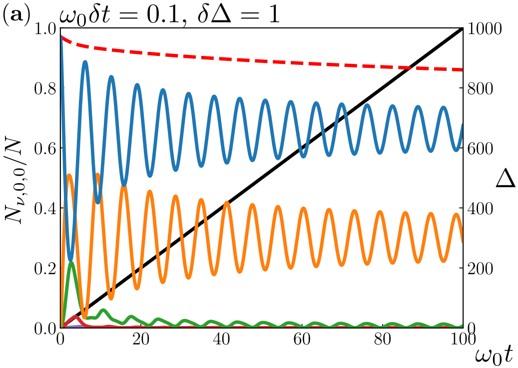
Nonequilibrium dynamics (dilute gas)

Compute instantaneous mode occupations:

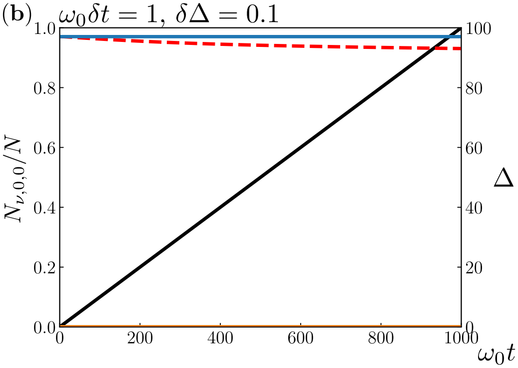
$$N_\alpha(t) = \mathrm{Tr}\left(\hat{\rho}(t)\,\hat{b}_\alpha^{S\dagger}(t)\hat{b}_\alpha^{S}(t)\right)$$



"Non-adiabatic" quench



"Adiabatic" quench



Thin-shell limit: 2D quantum bubbles

Low-temperature system tightly confined to surface of a 2-sphere[9]

$$\hat{H}_{\mathrm{2D}}(t) = \int_{S^2} d\Omega \left[\hat{\psi}^\dagger \left(\frac{L^2}{2mR(t)^2} + V(t) \right) \hat{\psi} + \frac{\lambda(t)}{2} \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} \right]$$

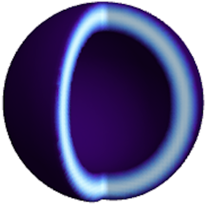
Analogy between linearized EOM for phase fluctuations and a massless relativistic scalar field:[10]

$$\Box \delta \hat{\varphi} = 0$$

$$\Box \equiv \frac{1}{\sqrt{|g|}} \partial_\mu \left(\sqrt{|g|} g^{\mu\nu} \partial_\nu \right)$$

$$g_{\mu\nu}(t, \theta, \phi) \equiv \text{const.} \left(\frac{\rho_c}{m c_s(t) R(t)^2} \right)^2 \begin{pmatrix} c_s(t)^2 & 0 & 0 \\ 0 & -R(t)^2 & 0 \\ 0 & 0 & -R(t)^2 \sin^2\theta \end{pmatrix}$$

Thin shell



---

[9] Tononi, Lewenstein, and Santos, "Geometric filtering effect in expanding Bose-Einstein condensate shells", 2025.

[10] Barceló, Liberati, and Visser, "Analogue Gravity", 2011.

Acknowledgements


T. Estrampes


G. Müller


N. Gaaloul

Theory

Experiment

Leibniz
Universität
Hannover


M. Misslisch


S. Gill


C. Garcion

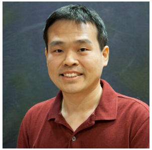

E. M. Rasel

université
PARIS-SACLAY


E. Charron


Z. Mehdi

Australian
National
University


K. Sun

WASHINGTON STATE
UNIVERSITY


S. Vishveshwara

UNIVERSITY OF
ILLINOIS
URBANA-CHAMPAIGN


A. Beregi


J.B. Gerent


N. Lundblad

BATES

DLR

NASA