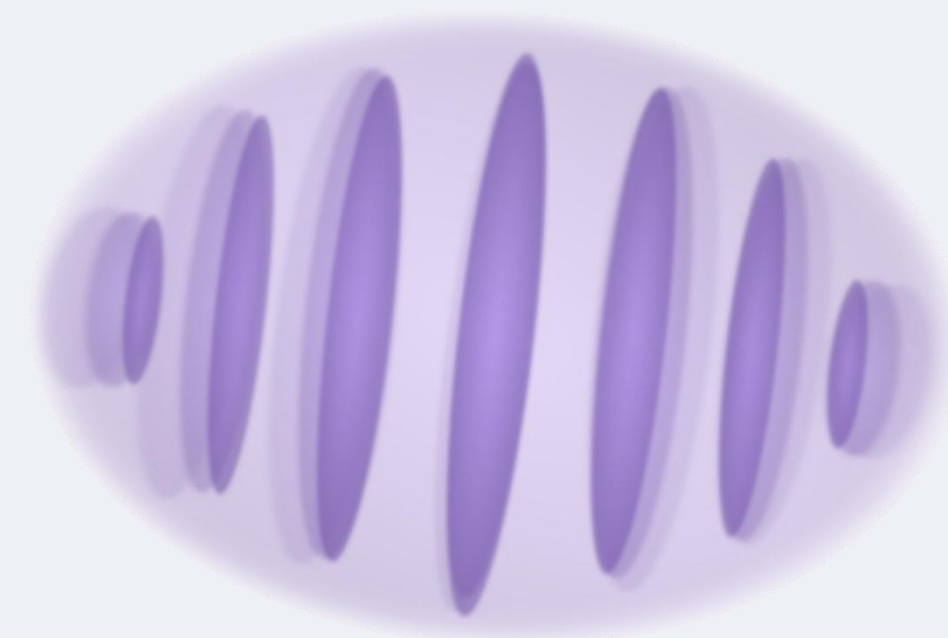


Engineering unconventional superfluids with ultracold bosonic mixtures

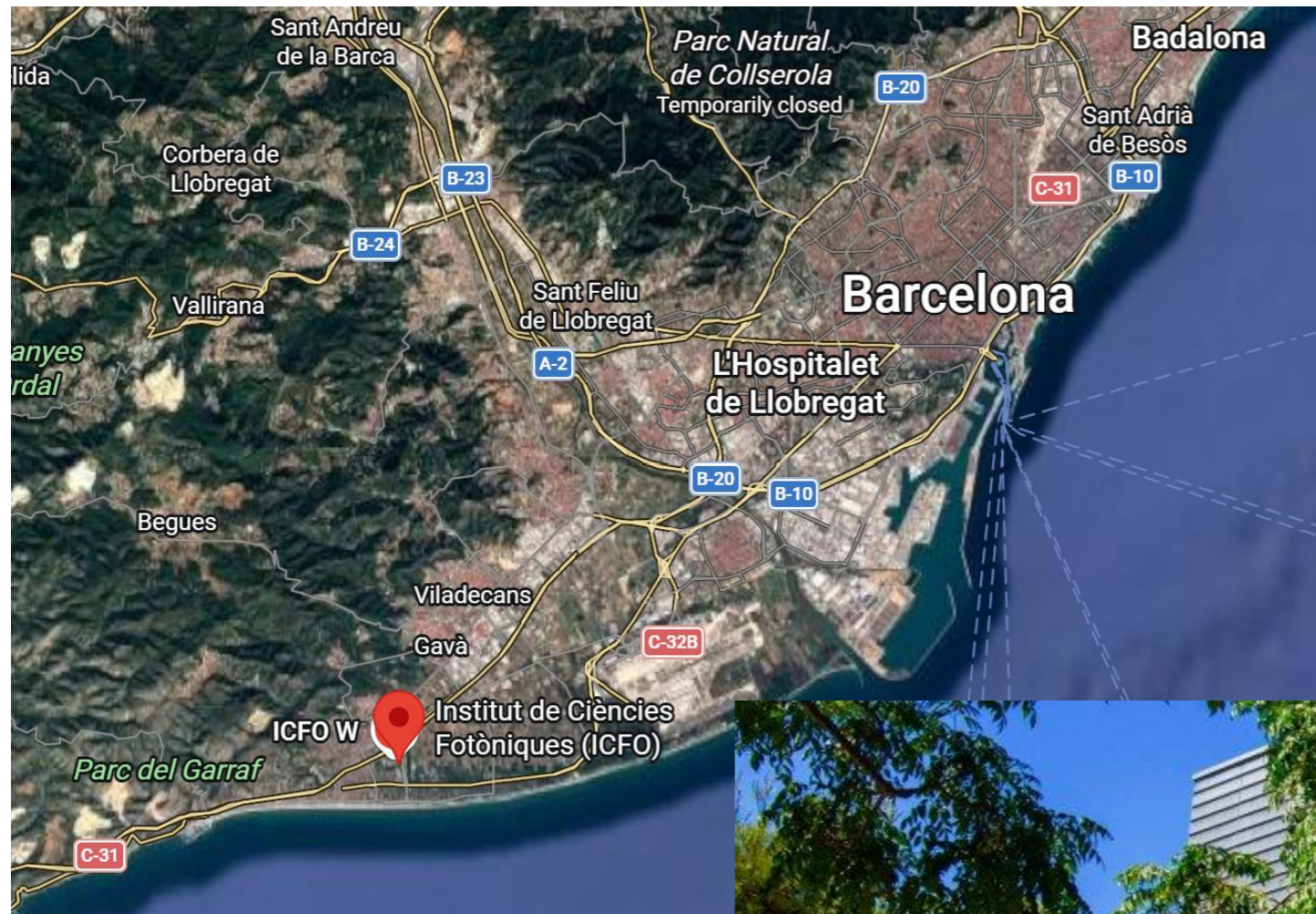


Sarah Hirthe

Group of Leticia Tarruell
ICFO (Barcelona, Spain)



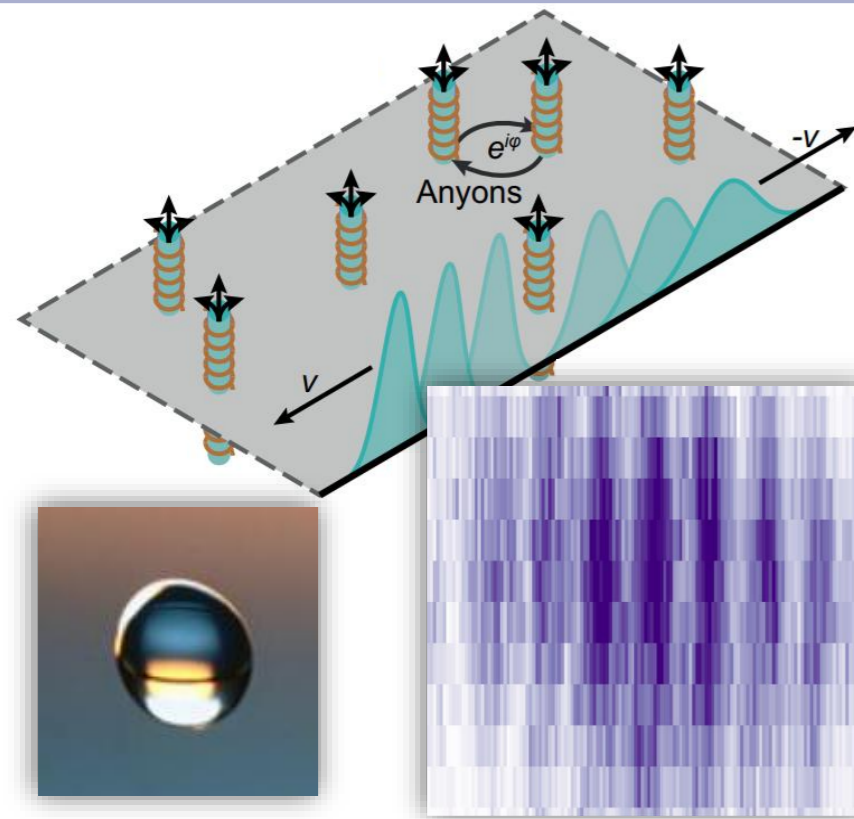
Frontiers in Ultracold Quantum Gases
Maó 2026



Leticia
Tarruell



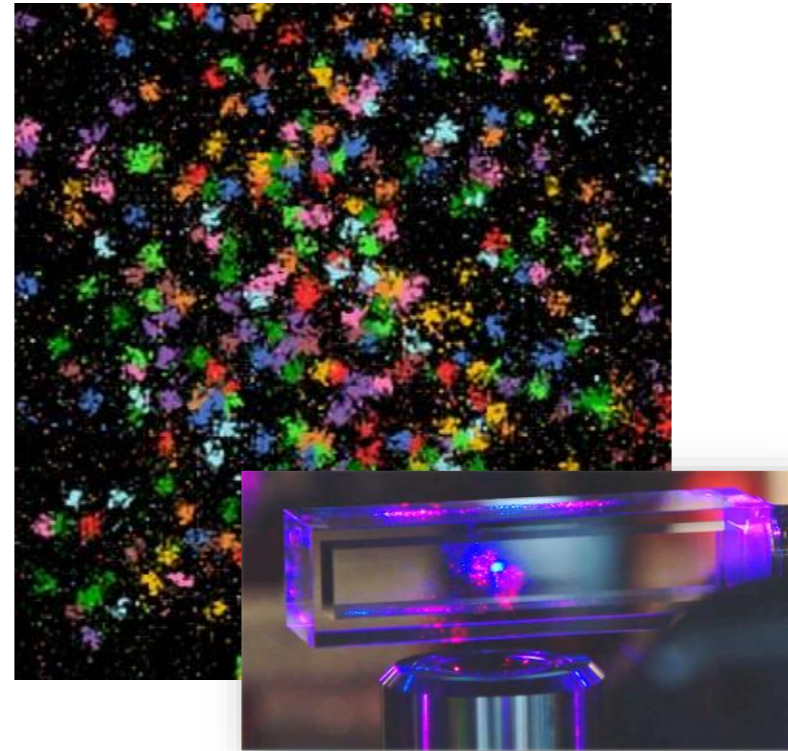
Potassium lab



Potassium spin mixtures

Cabrera et al., *Science* **359** (2018)
 Frölian et al., *Nature* **608** (2022)
 Chisholm et al., *Science* **391** (2026)
 ... and more

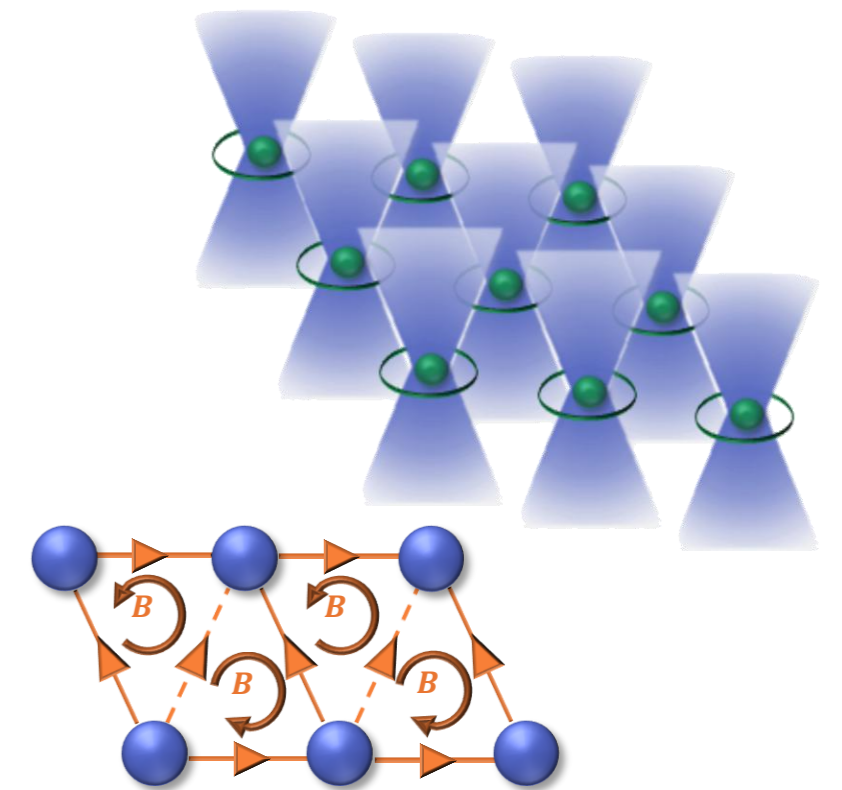
Strontium lab



Quantum gas microscope

Höschele et al., *PR Applied* **19** (2023)
 Buob et al., *PRX Quantum* **5** (2024)
 Gas-Ferrer et al., *arXiv:2603.05478*

Rydberg lab



Strontium Rydberg tweezers

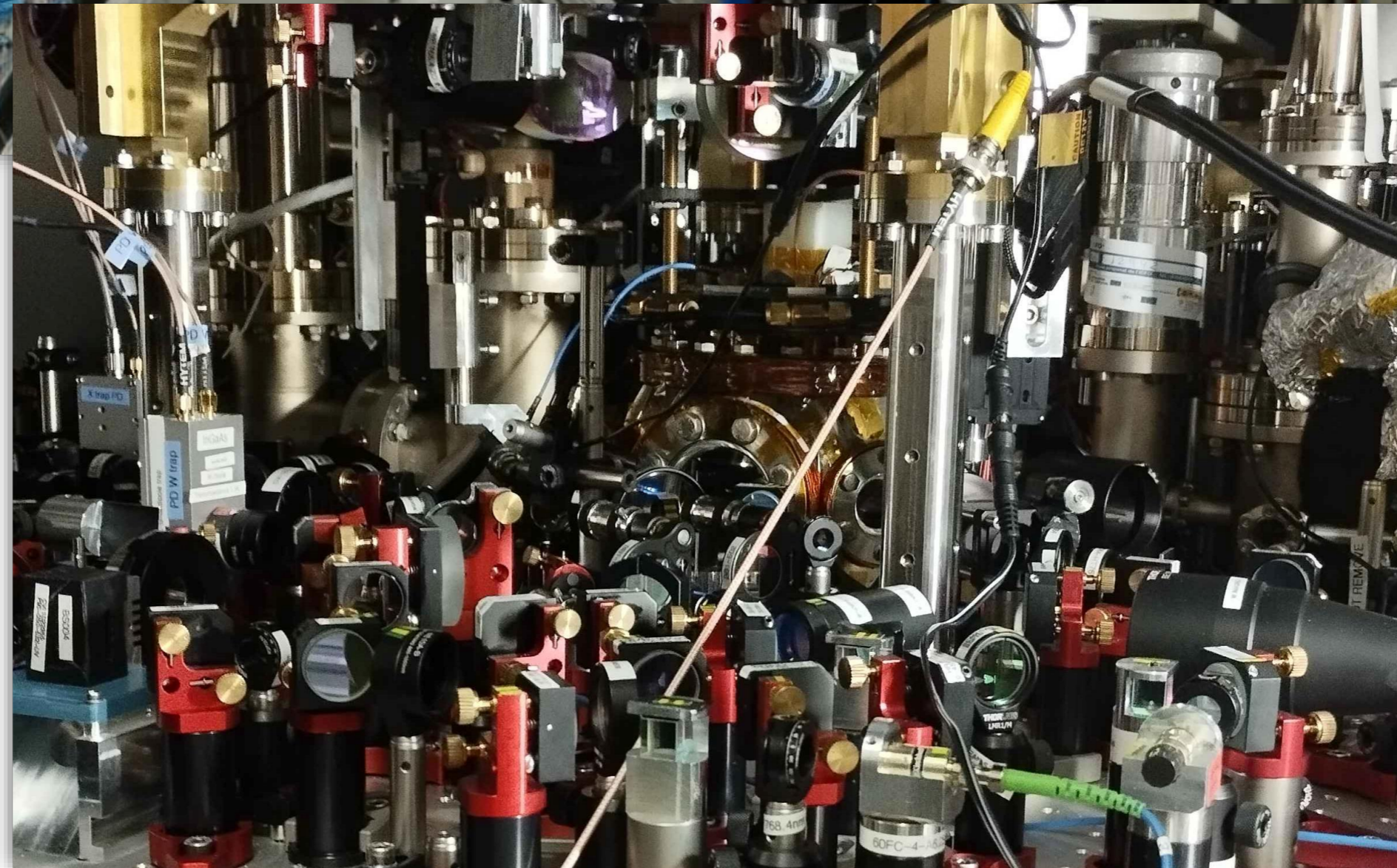
I. Bosonic mixtures with tunable interactions

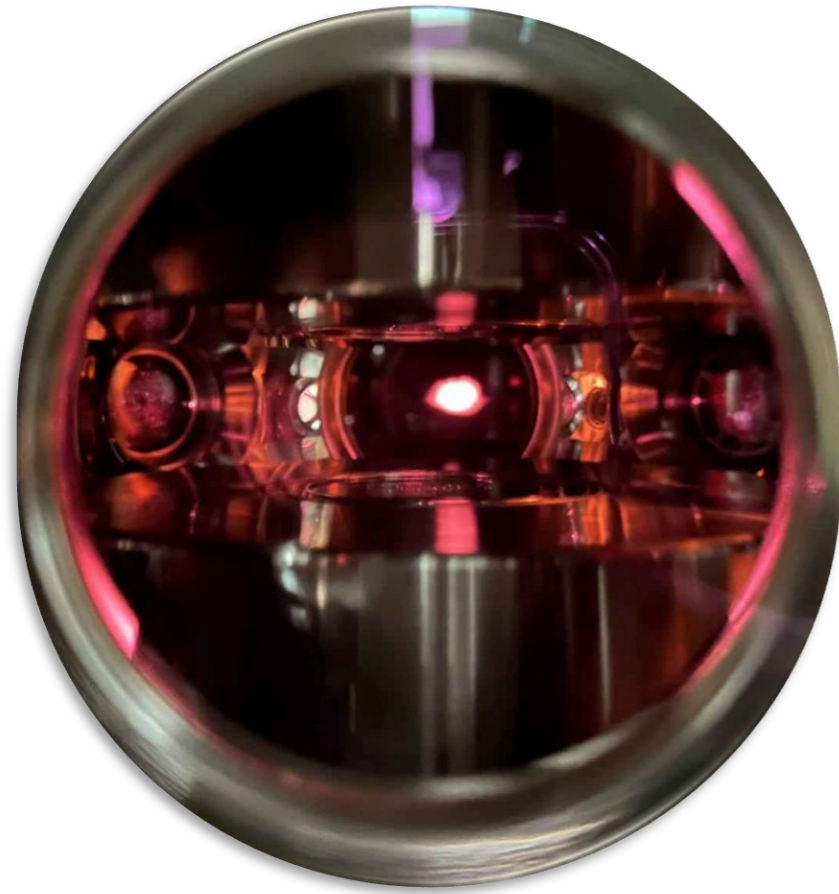
Quantum liquid droplets

II. Raman-coupled mixtures

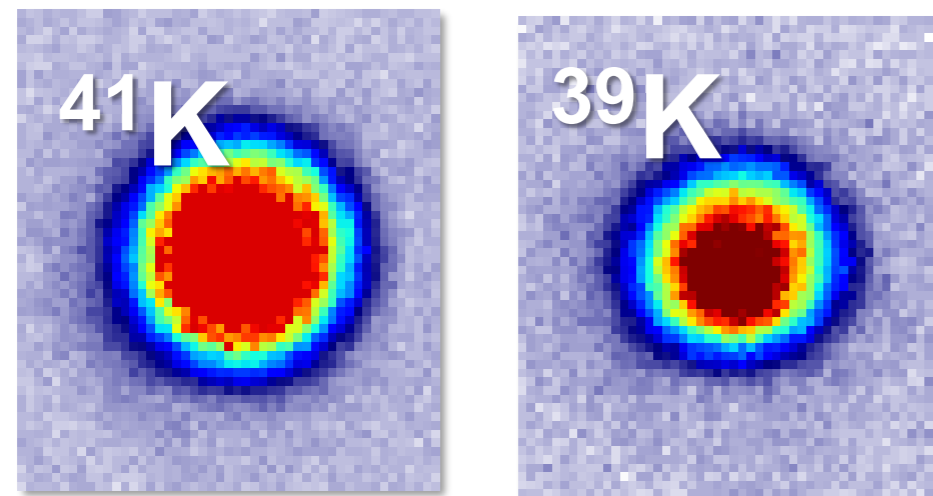
Supersolids

Chiral solitons

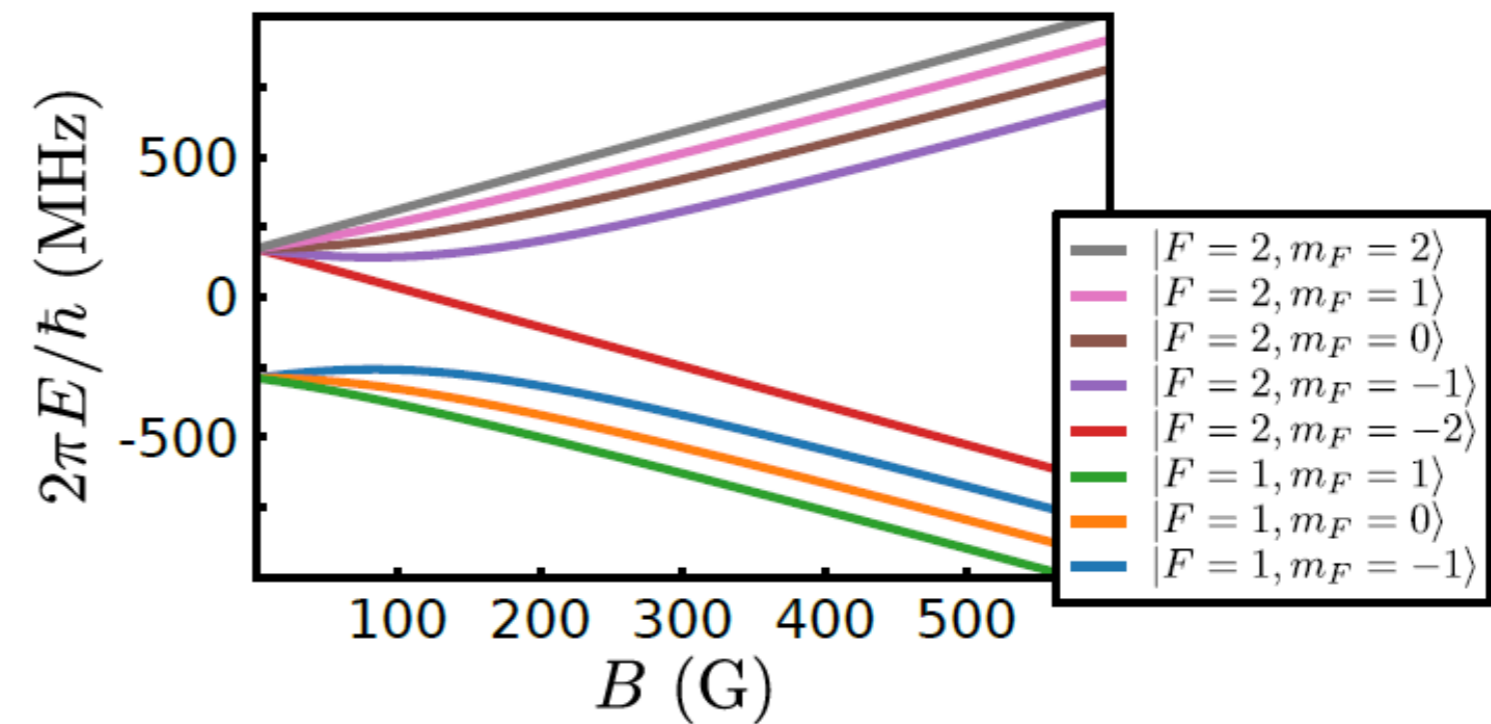
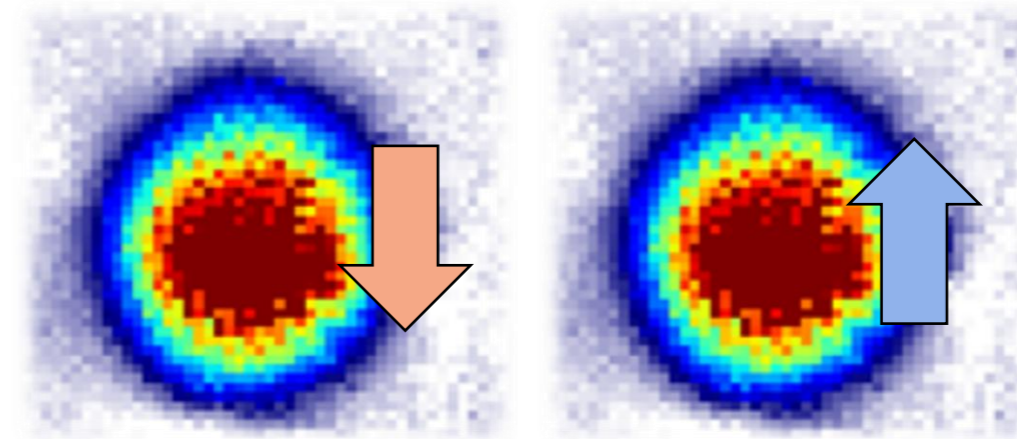




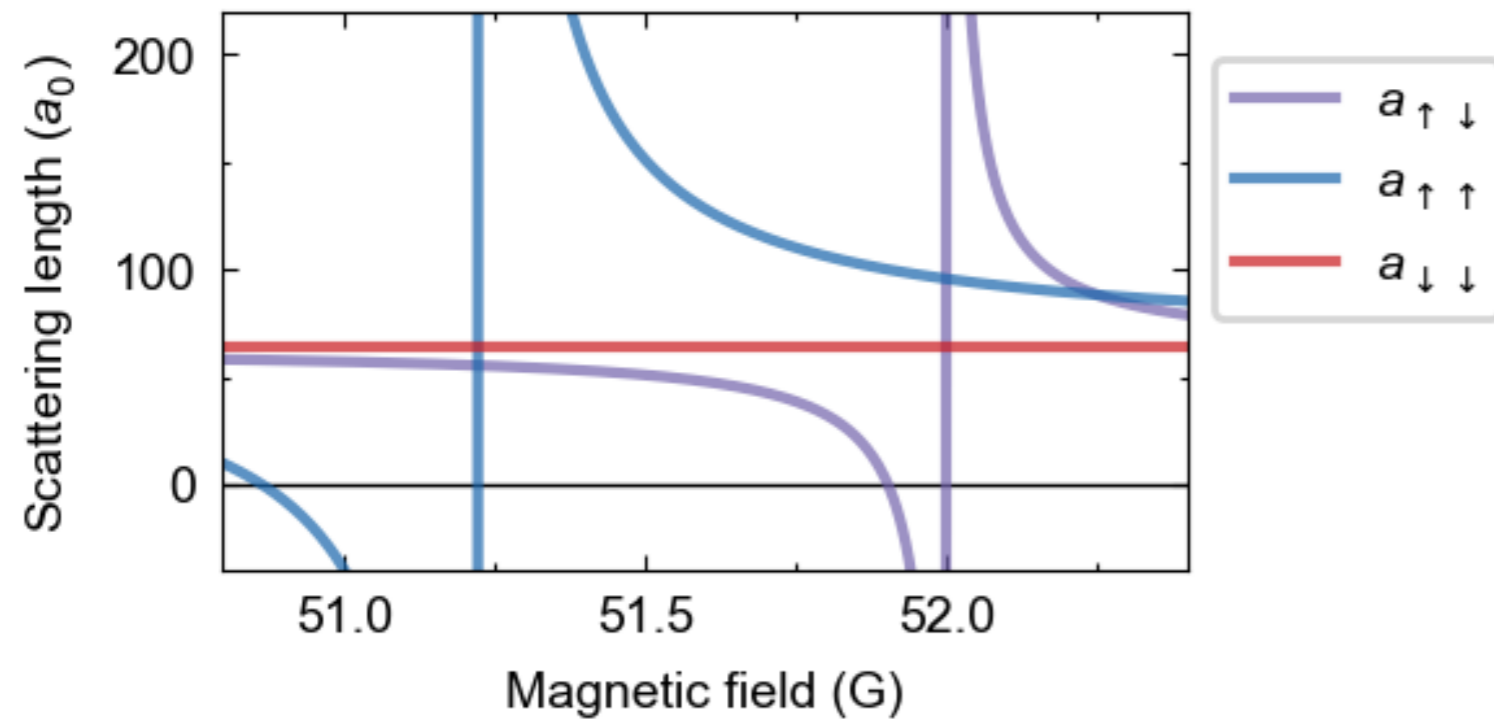
Two bosonic isotopes



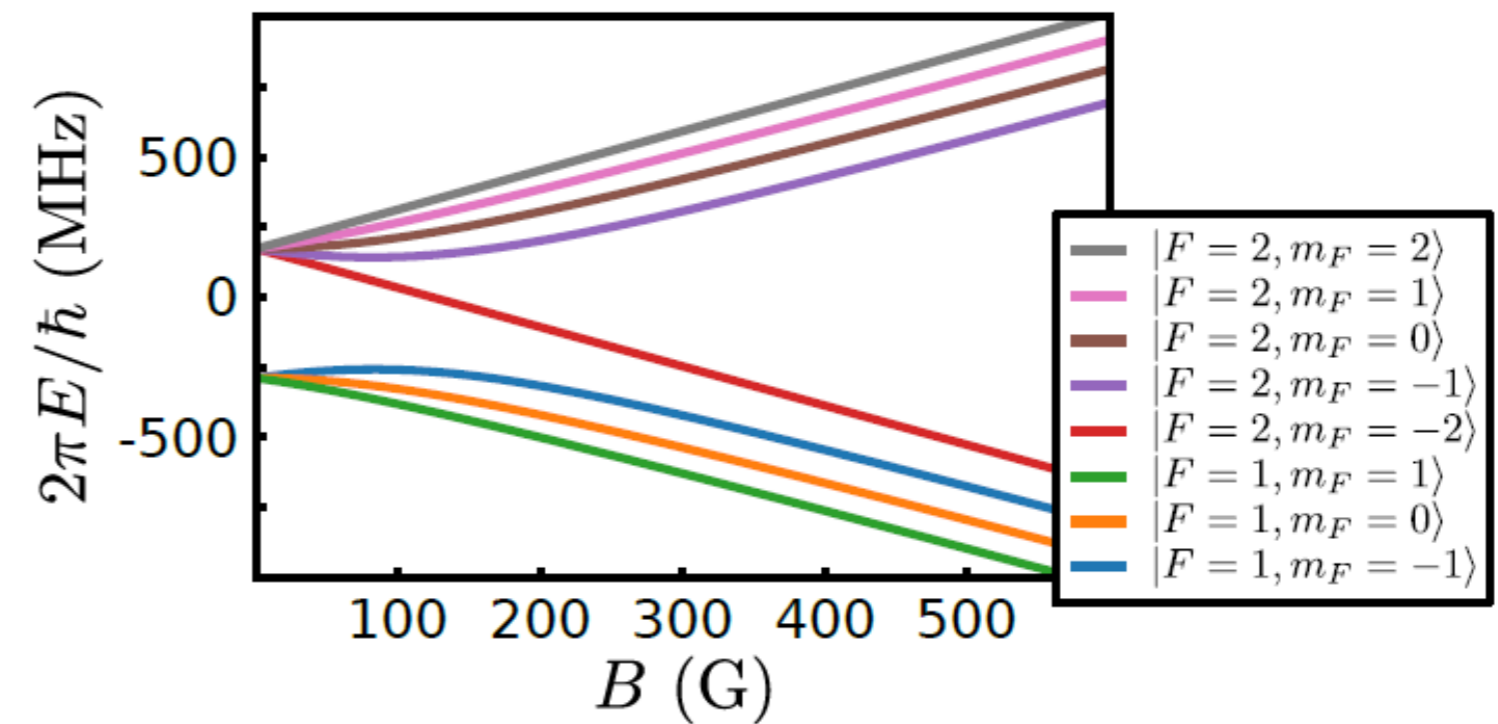
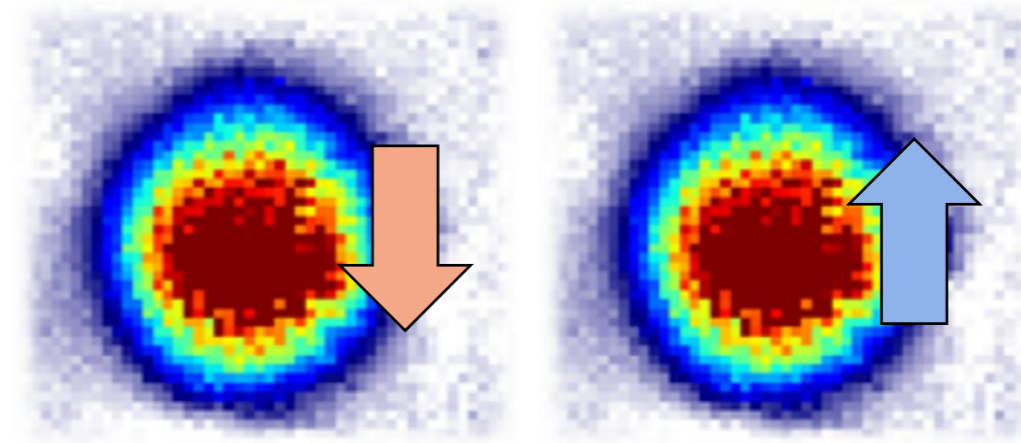
^{41}K spin mixtures



Tunable interactions

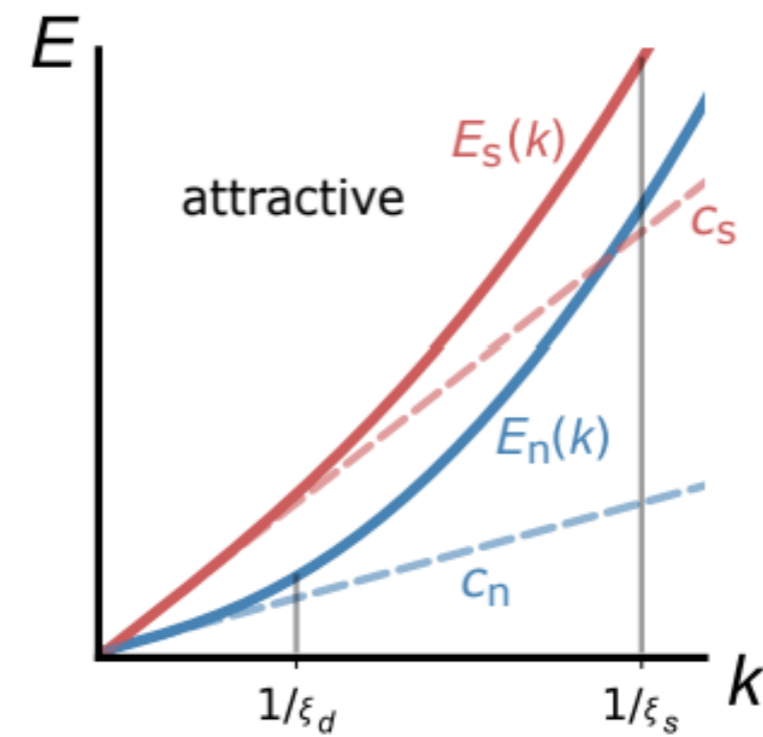
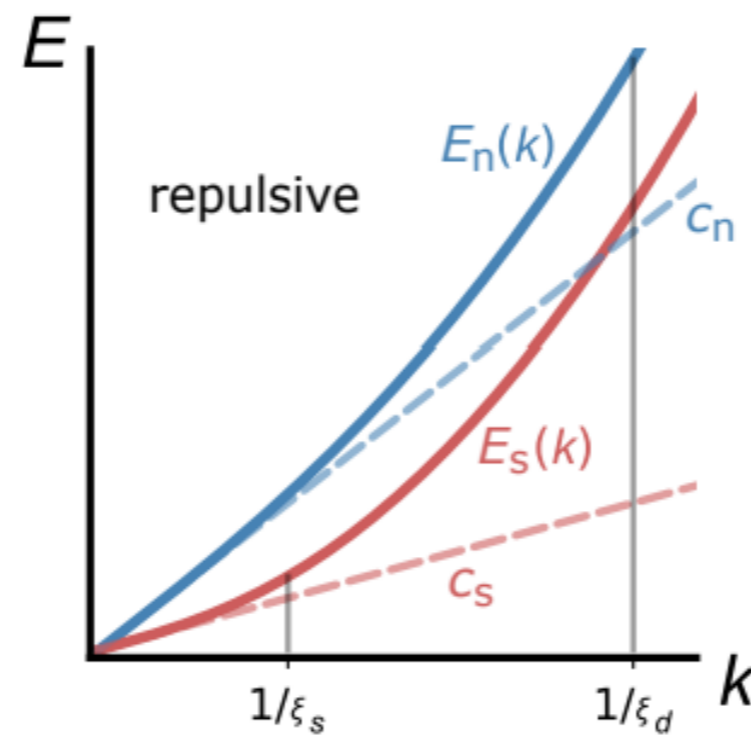
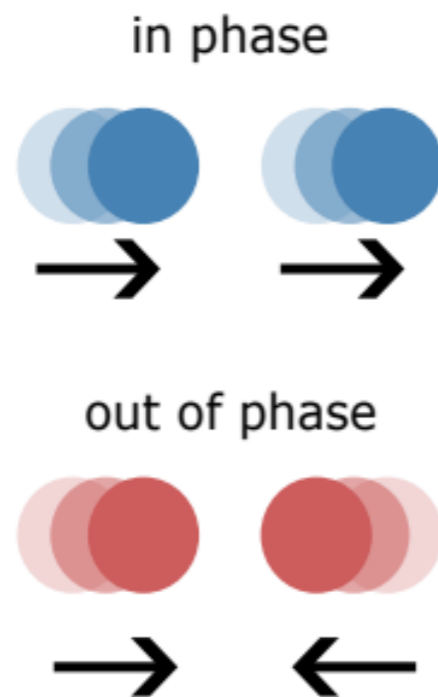


^{41}K spin mixtures

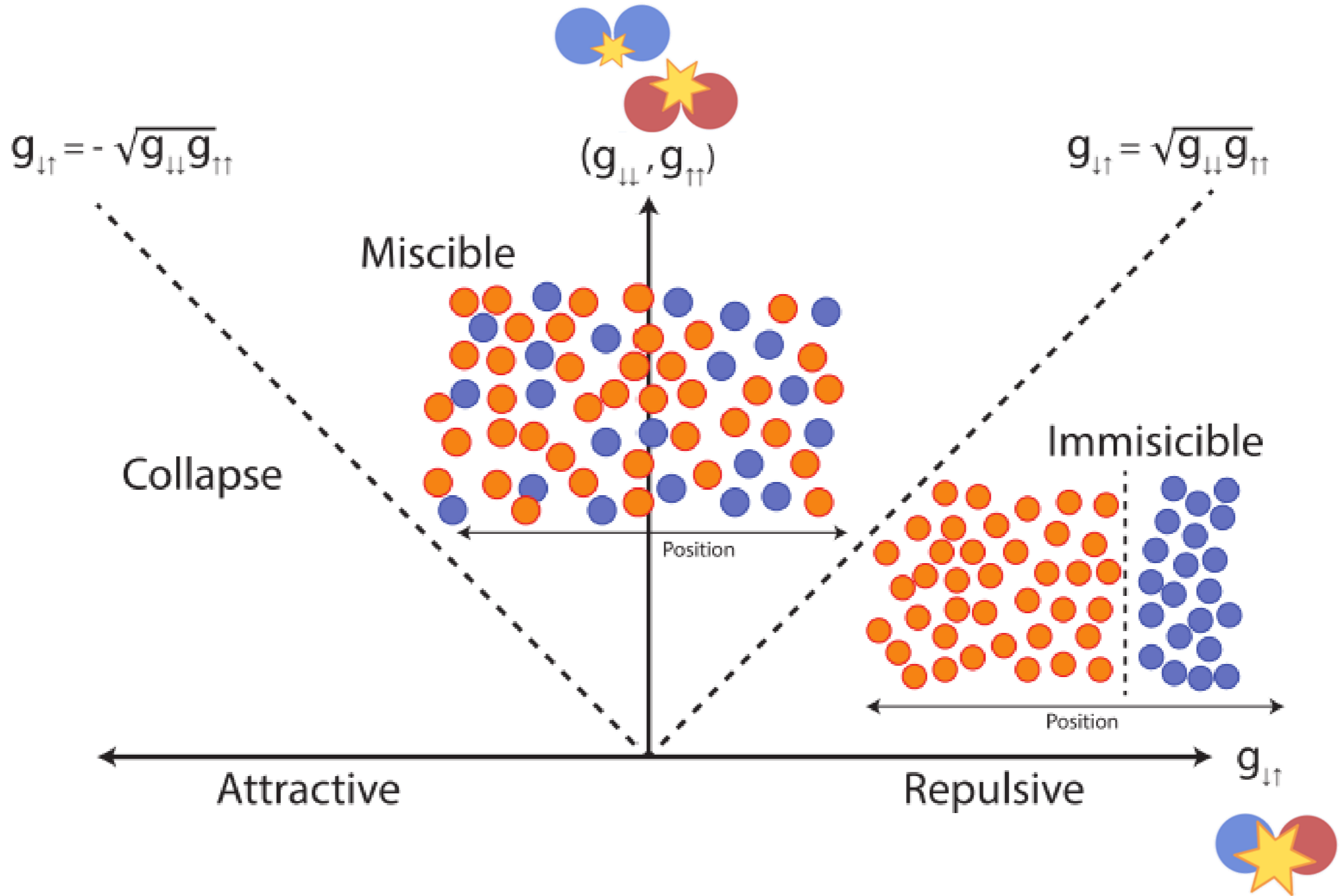


$$g_{\uparrow\uparrow} = g_{\downarrow\downarrow} = \bar{g}$$

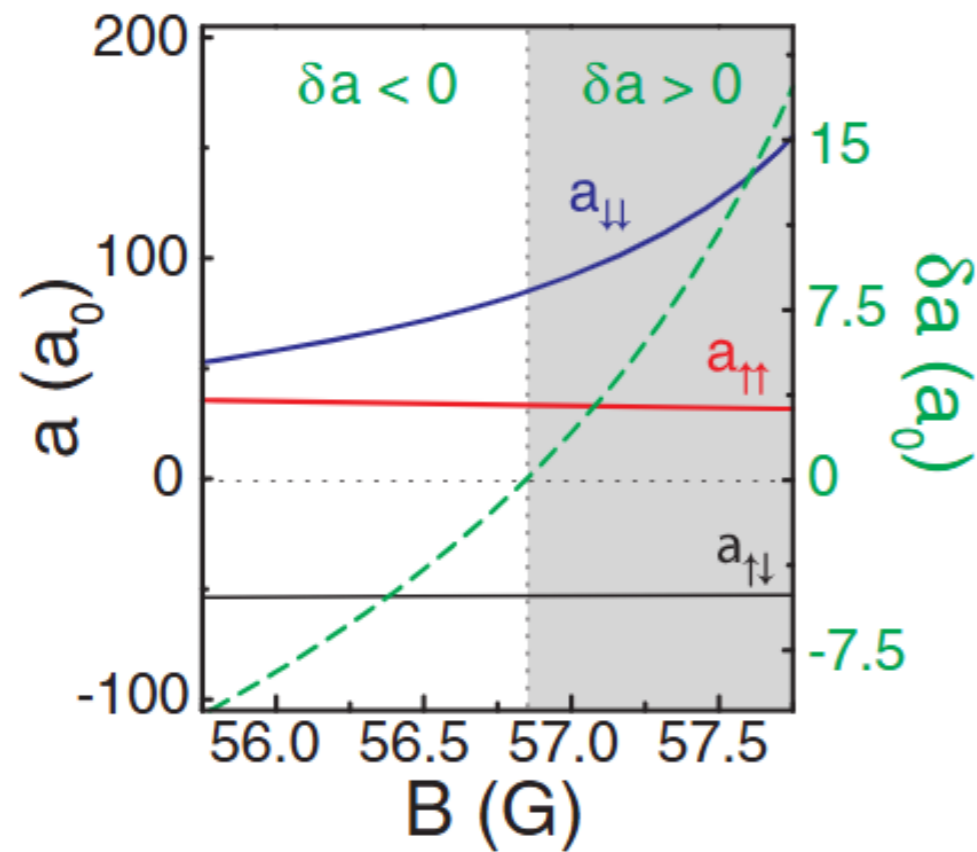
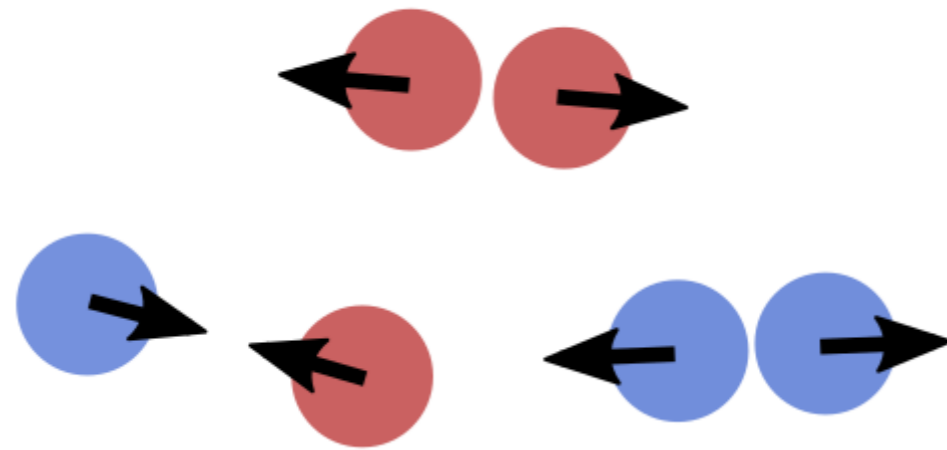
$$E(k)_{\pm} = \sqrt{\frac{\hbar^2 k^2}{2m} \left(\frac{\hbar^2 k^2}{2m} + 2mc_{\pm}^2 \right)}, \quad c_n = \sqrt{\frac{(\bar{g} + g_{\uparrow\downarrow})n}{2m}} \quad \text{and} \quad c_s = \sqrt{\frac{(\bar{g} - g_{\uparrow\downarrow})n}{2m}}$$



Mean field phase diagram

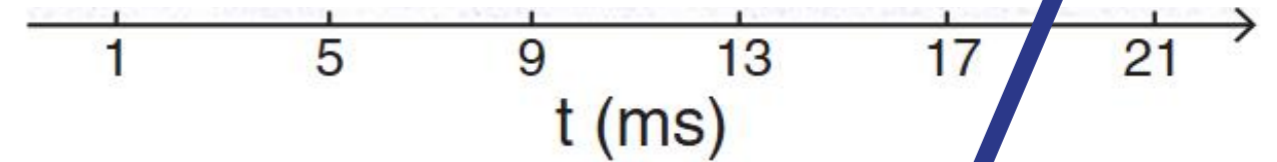
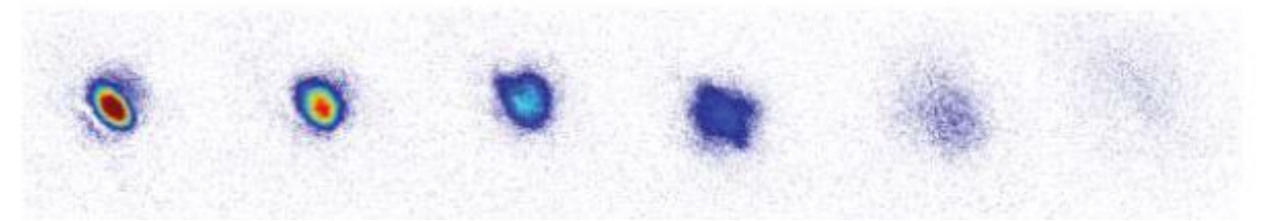


Cabrera, PhD thesis (2018)



$$\delta a = a_{\uparrow\downarrow} + \sqrt{a_{\uparrow\uparrow}a_{\downarrow\downarrow}}$$

$\delta a > 0$
Gas $\uparrow\downarrow$



Quantum liquid droplets!



Beyond mean field effects!

Mean field energy basically cancels

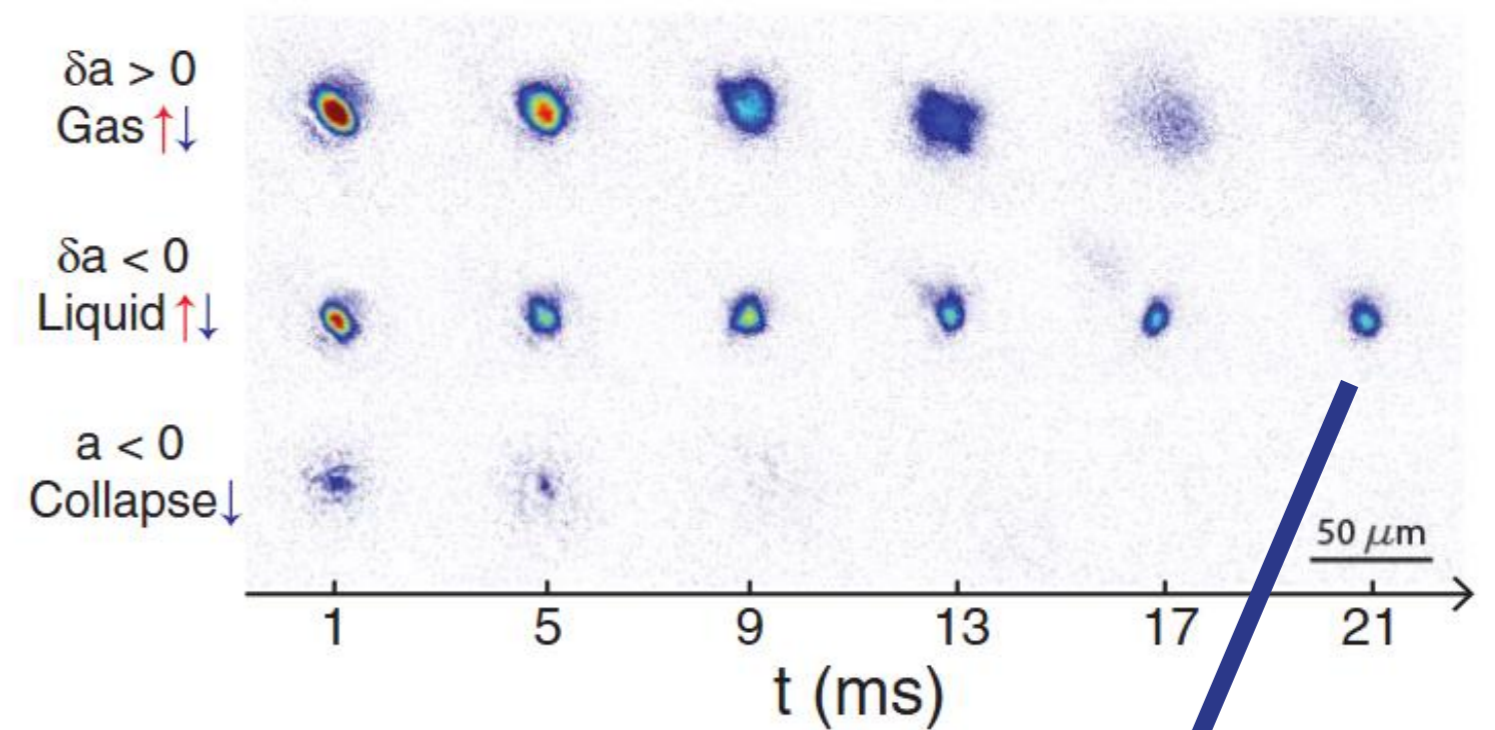
$$\mathcal{E}_{MF} = \frac{g + g_{\uparrow\downarrow}}{4} n^2 \quad g_{\uparrow\uparrow} = g_{\downarrow\downarrow} = g$$

Beyond mean field term relevant!

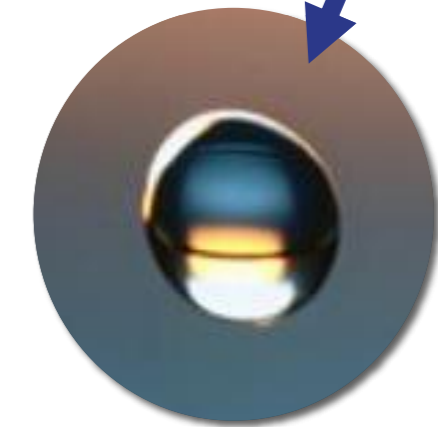
$$\mathcal{E}_{LHY} = \frac{8m^{3/2}}{15\pi^2\hbar^3} \left(\frac{n}{2}\right)^{5/2} \left[(g + g_{\uparrow\downarrow})^{5/2} + (g - g_{\uparrow\downarrow})^{5/2} \right]$$

Lee-Huang-Yang term

Phase stabilized by quantum fluctuations



Quantum liquid droplets!



Beyond mean field effects!

Mean field energy basically cancels

$$\mathcal{E}_{MF} = \frac{g + g_{\uparrow\downarrow}}{4} n^2 \quad g_{\uparrow\uparrow} = g_{\downarrow\downarrow} = g$$

Beyond mean field term relevant!

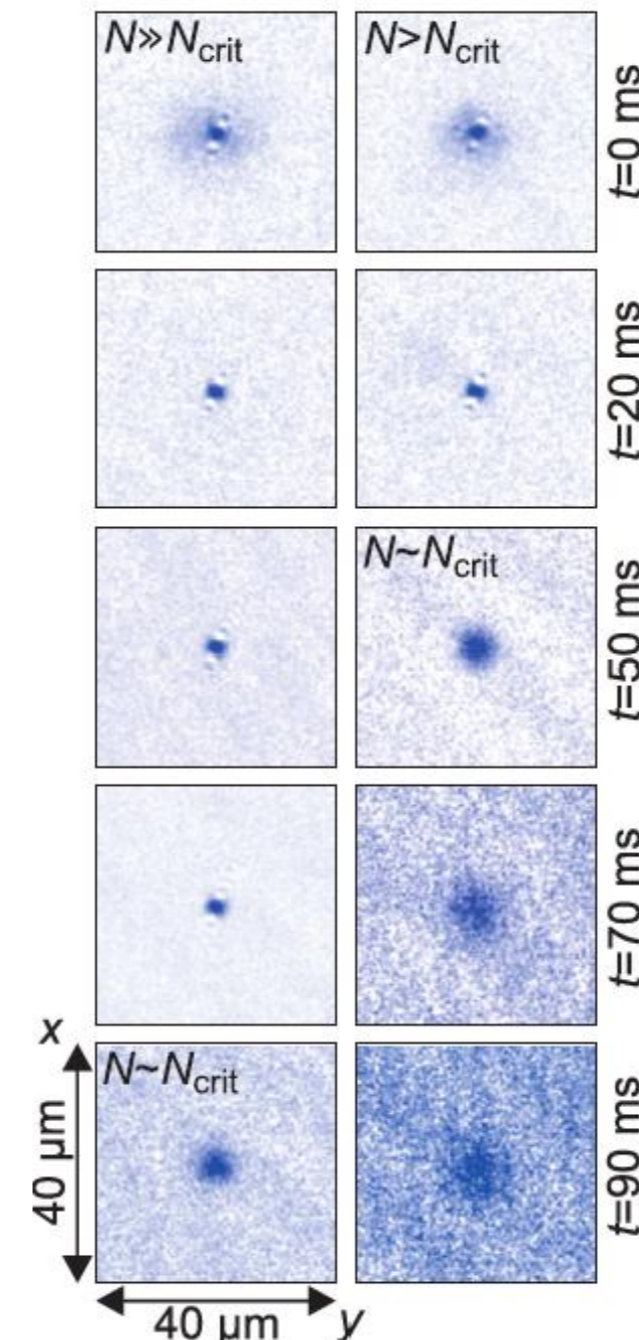
$$\mathcal{E}_{LHY} = \frac{8m^{3/2}}{15\pi^2\hbar^3} \left(\frac{n}{2}\right)^{5/2} \left[(g + g_{\uparrow\downarrow})^{5/2} + (g - g_{\uparrow\downarrow})^{5/2} \right]$$

Lee-Huang-Yang term

Phase stabilized by quantum fluctuations

First prediction for Bose mixtures

Petrov, *Phys. Rev. Lett.* **115** (2015).



First observation in dipolar quantum gases

Kadau et al., *Nature* **530** (2016).

I. Bosonic mixtures with tunable interactions

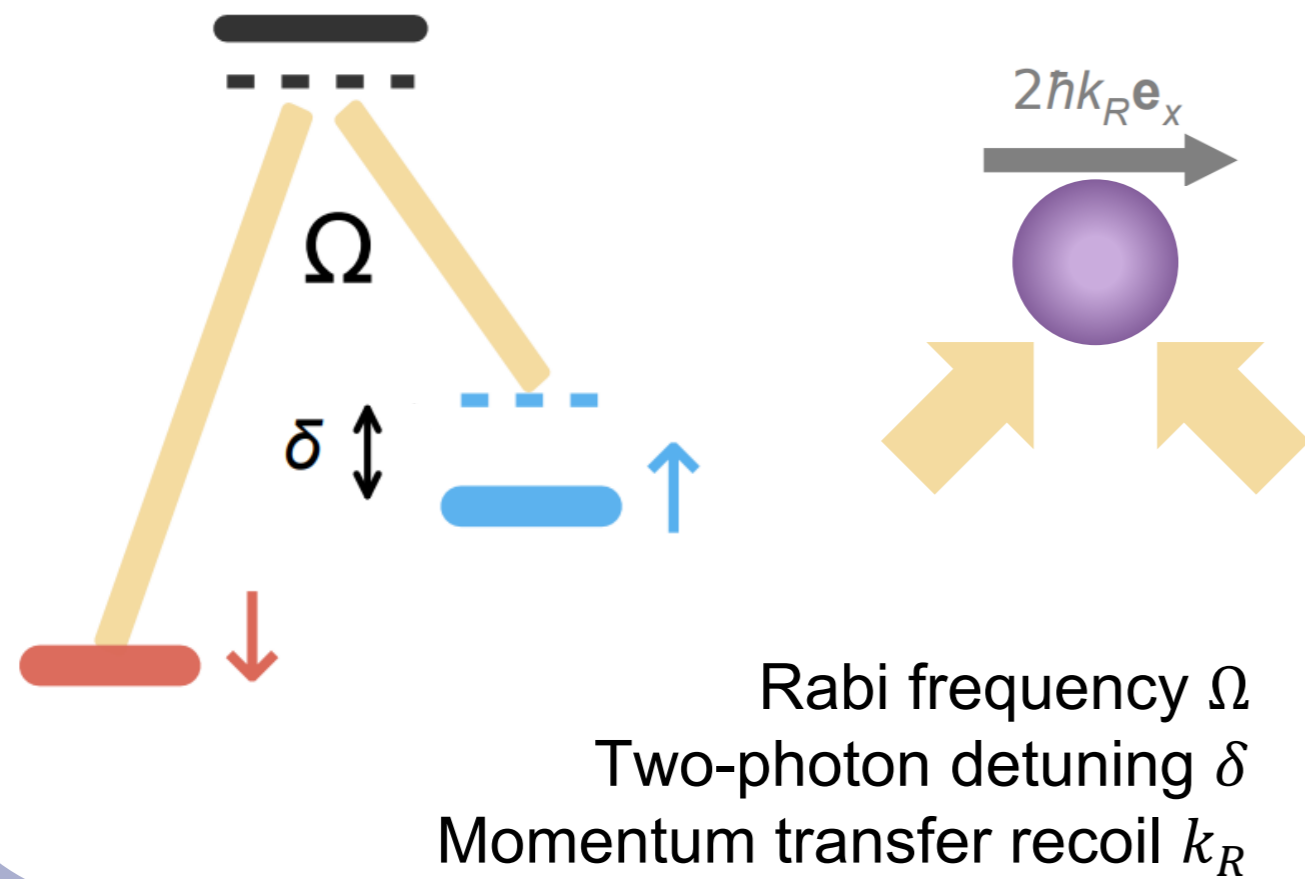
Quantum liquid droplets

II. Raman-coupled mixtures

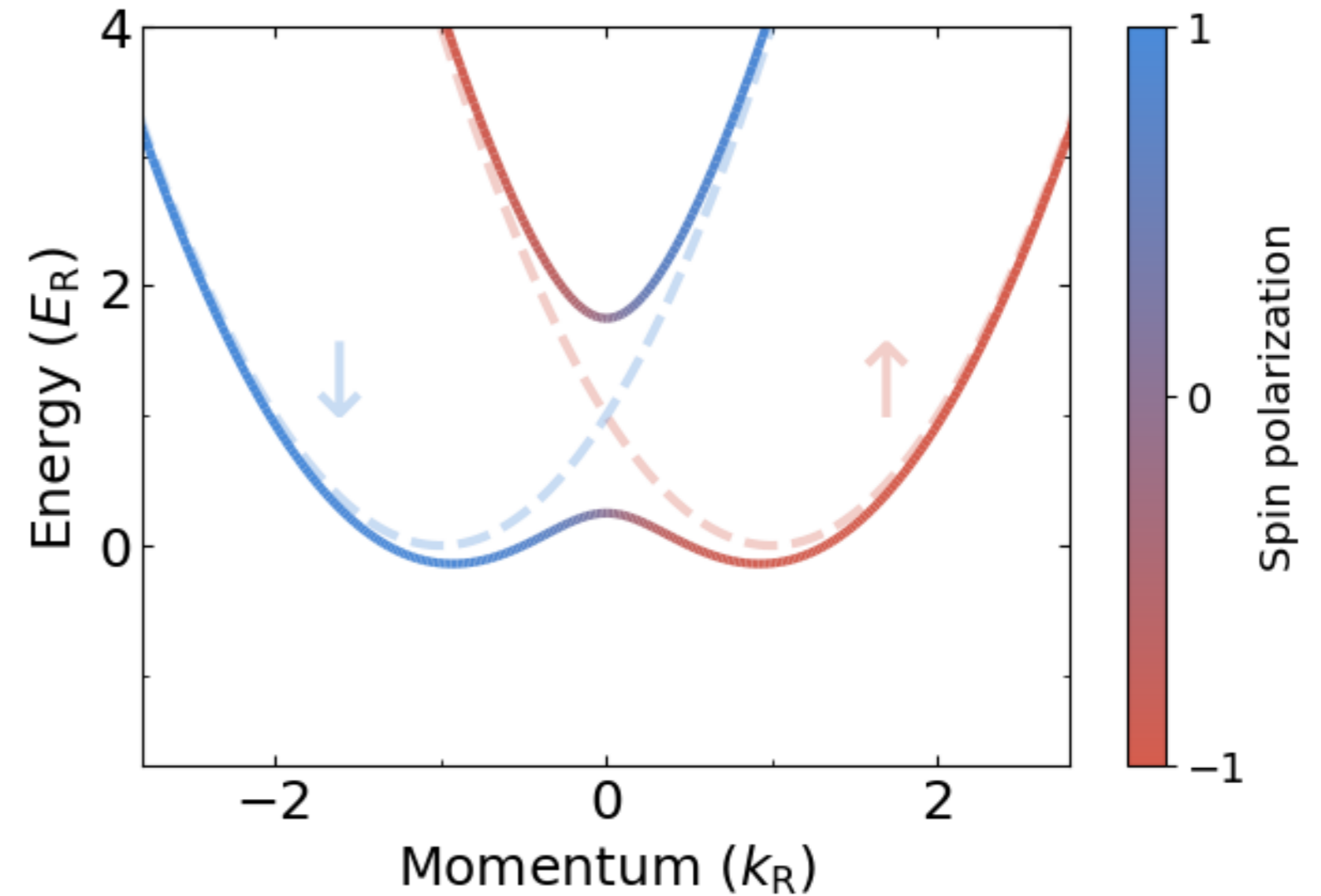
Supersolids

Chiral solitons

Raman coupling



Engineered dispersion relation

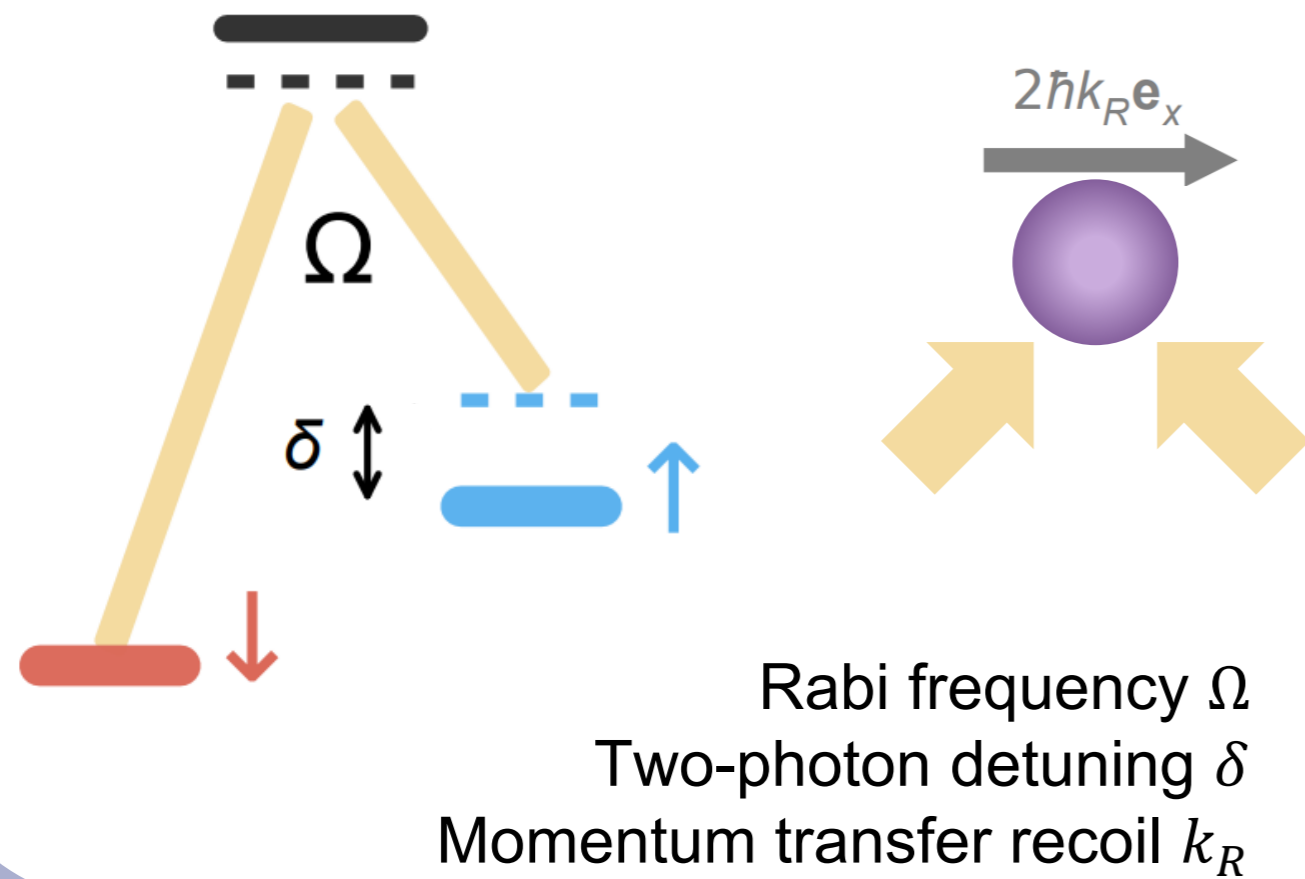


Lin, Jiménez-García, and Spielman,
Nature **471**, 83 (2011)

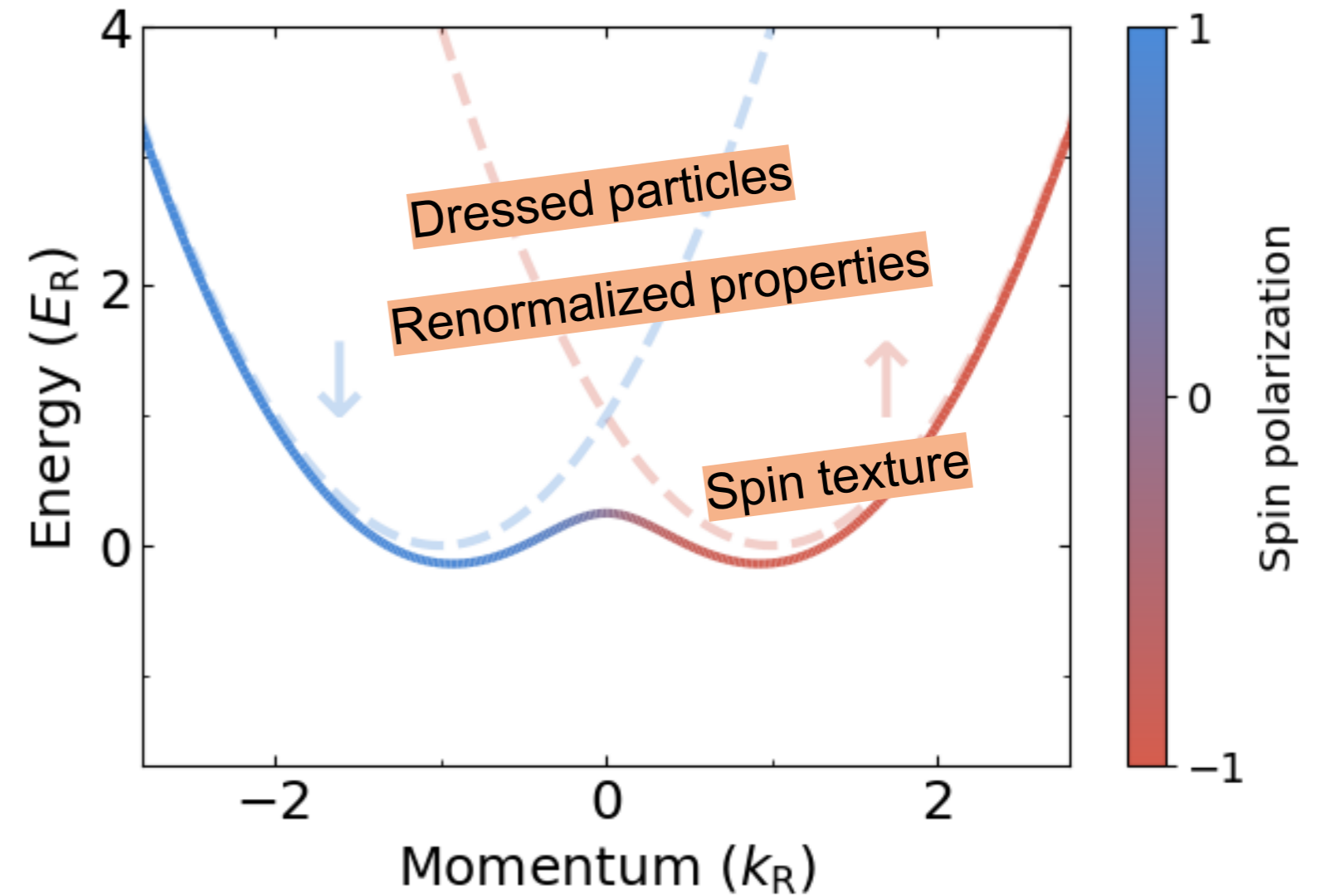
$$k_R = \frac{2\pi}{\lambda_R} \sin(\theta/2)$$

$$E_R = \hbar^2 k_R^2 / 2m$$

Raman coupling



Engineered dispersion relation

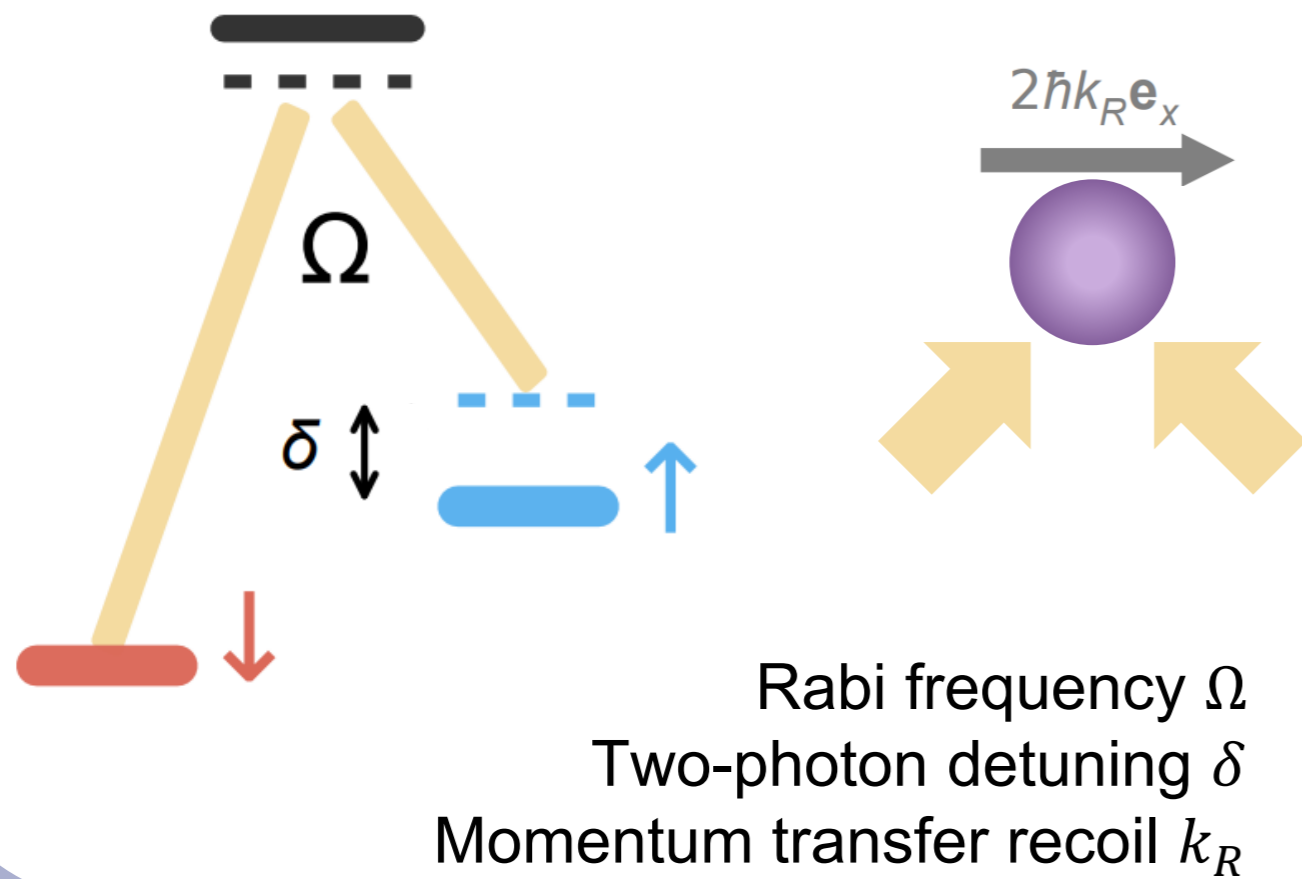


Lin, Jiménez-García, and Spielman,
Nature **471**, 83 (2011)

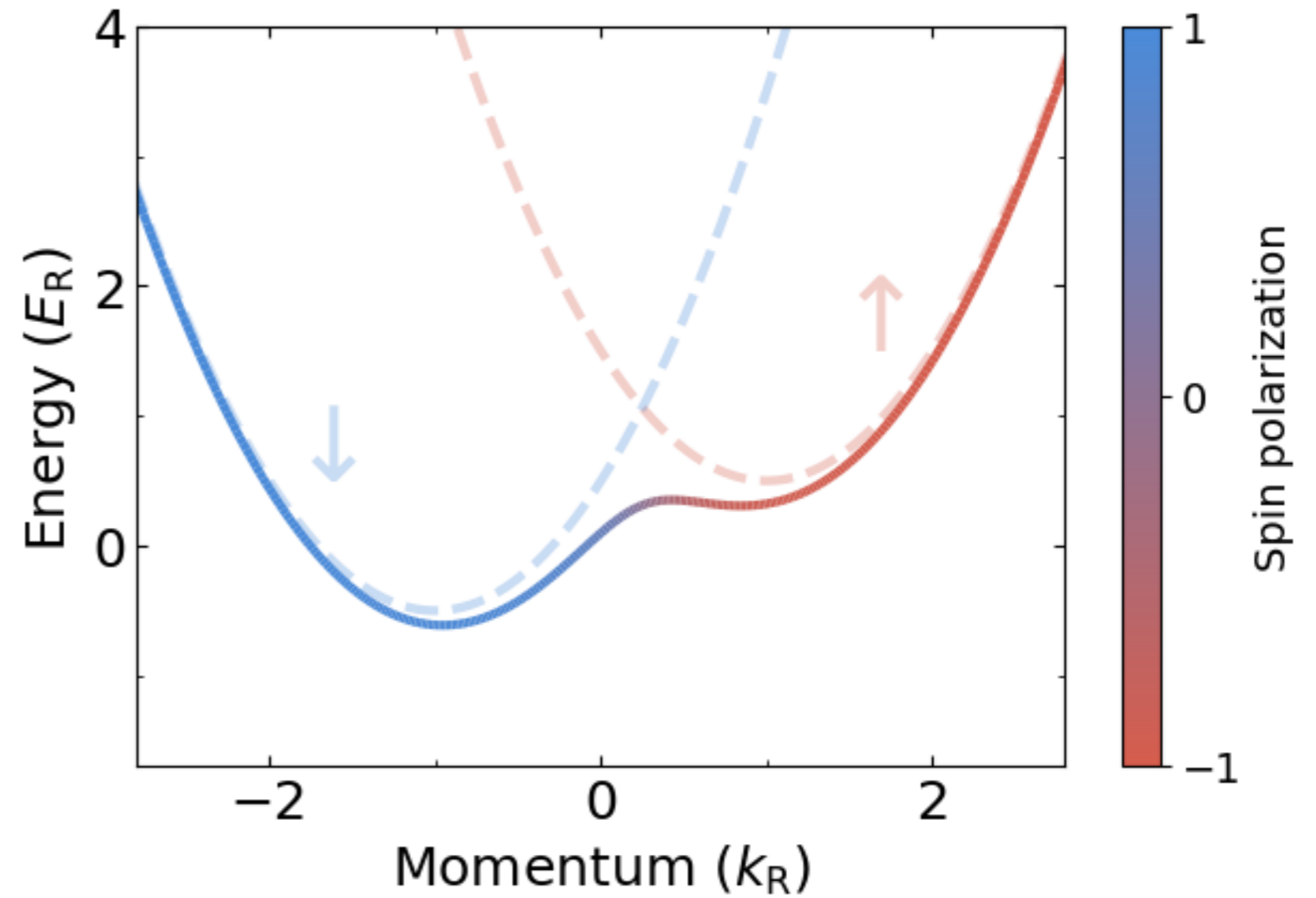
$$k_R = \frac{2\pi}{\lambda_R} \sin(\theta/2)$$

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Raman coupling



Engineered dispersion relation

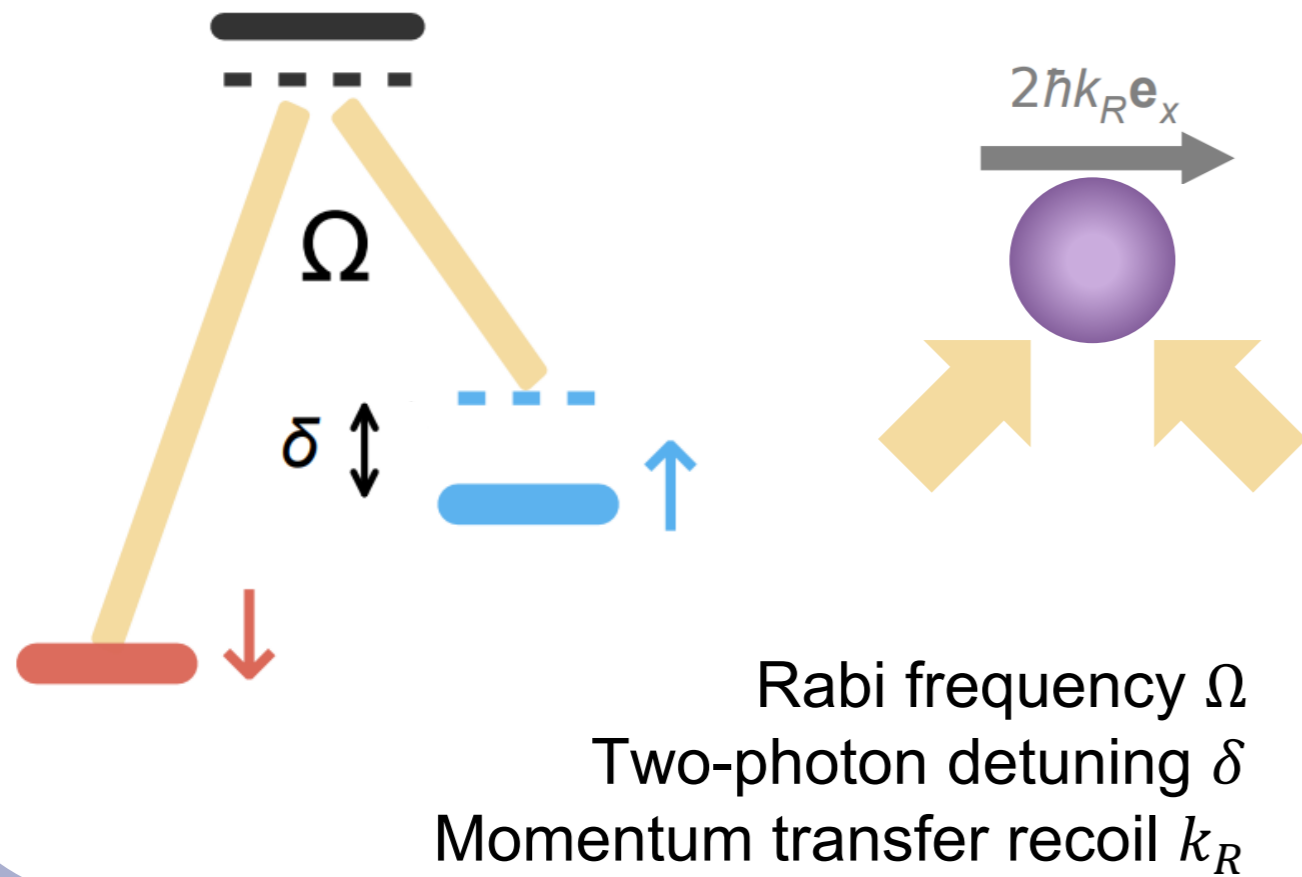


Lin, Jiménez-García, and Spielman,
Nature **471**, 83 (2011)

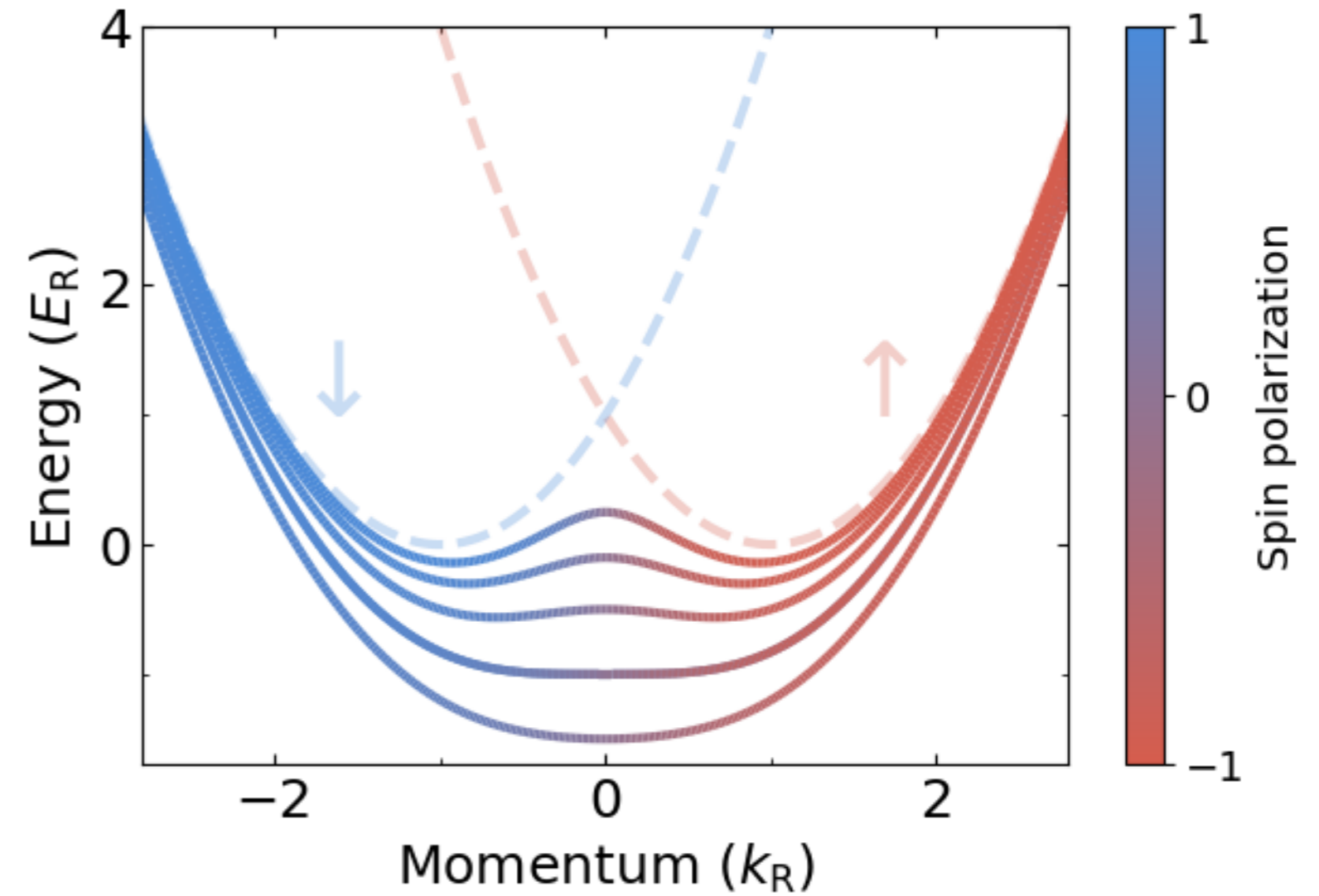
$$k_R = \frac{2\pi}{\lambda_R} \sin(\theta/2)$$

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Raman coupling



Engineered dispersion relation

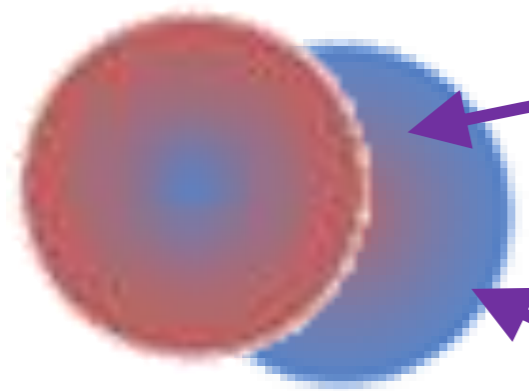


Lin, Jiménez-García, and Spielman,
Nature **471**, 83 (2011)

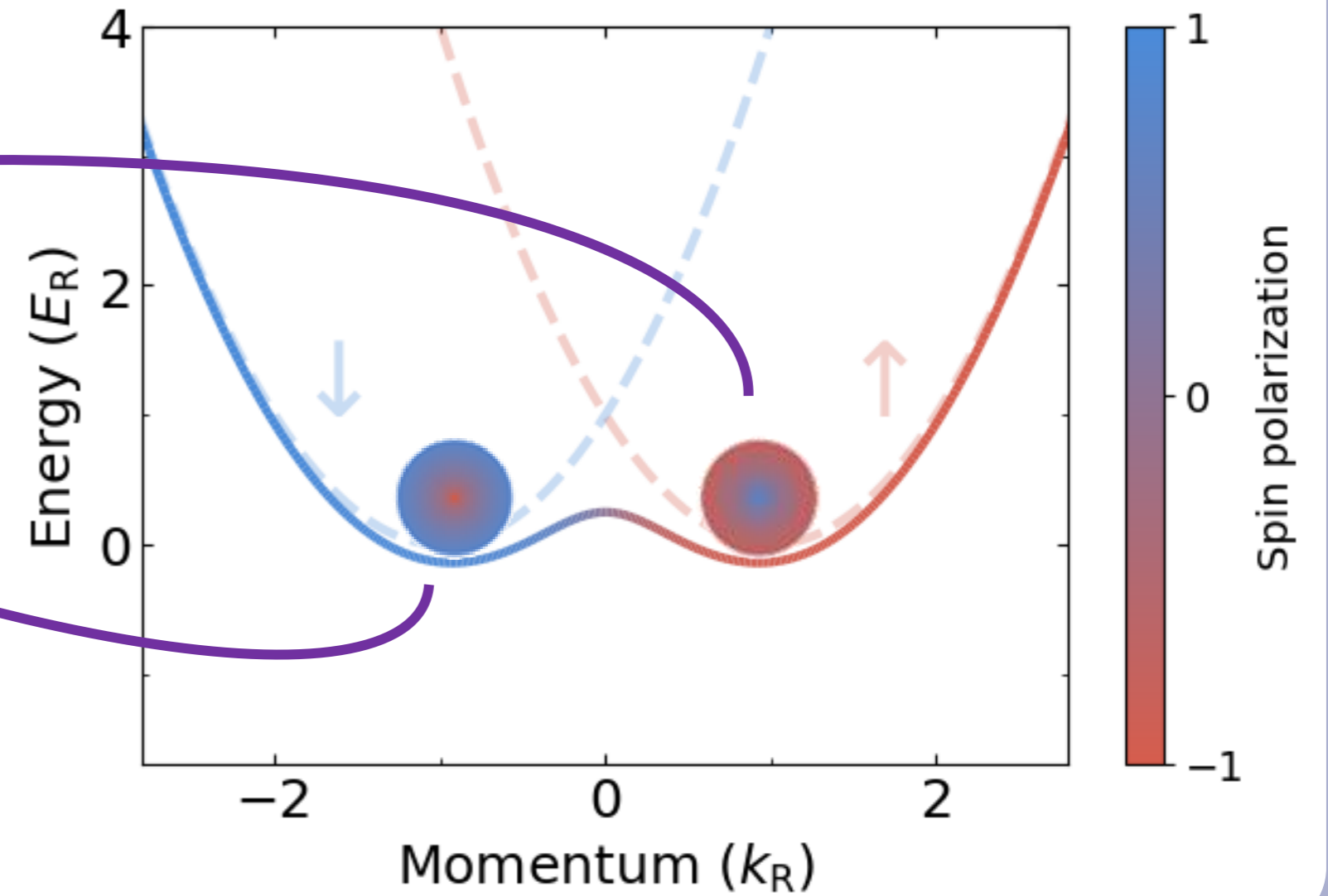
$$k_R = \frac{2\pi}{\lambda_R} \sin(\theta/2)$$

$$E_R = \hbar^2 k_R^2 / 2m$$

Matter wave interference

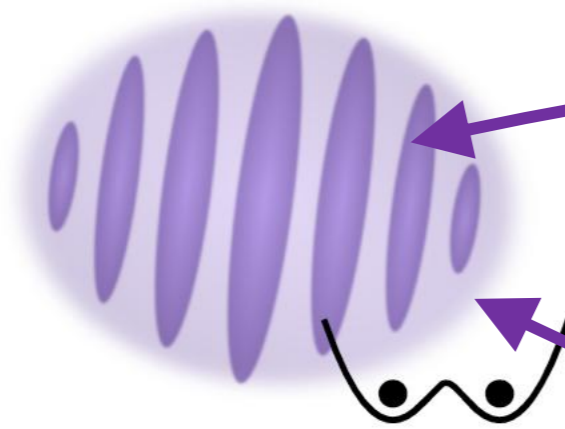


Spin-orbit coupling

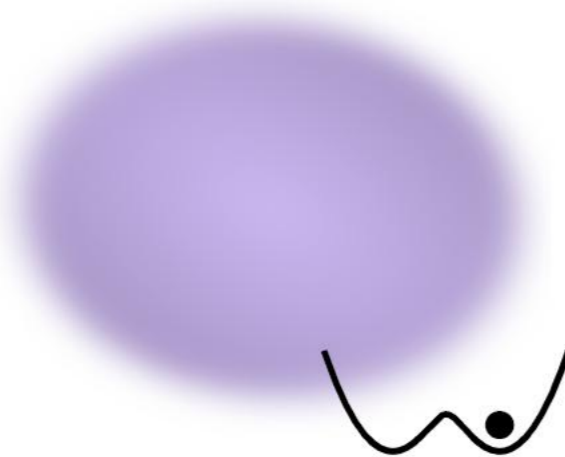


Matter wave interference

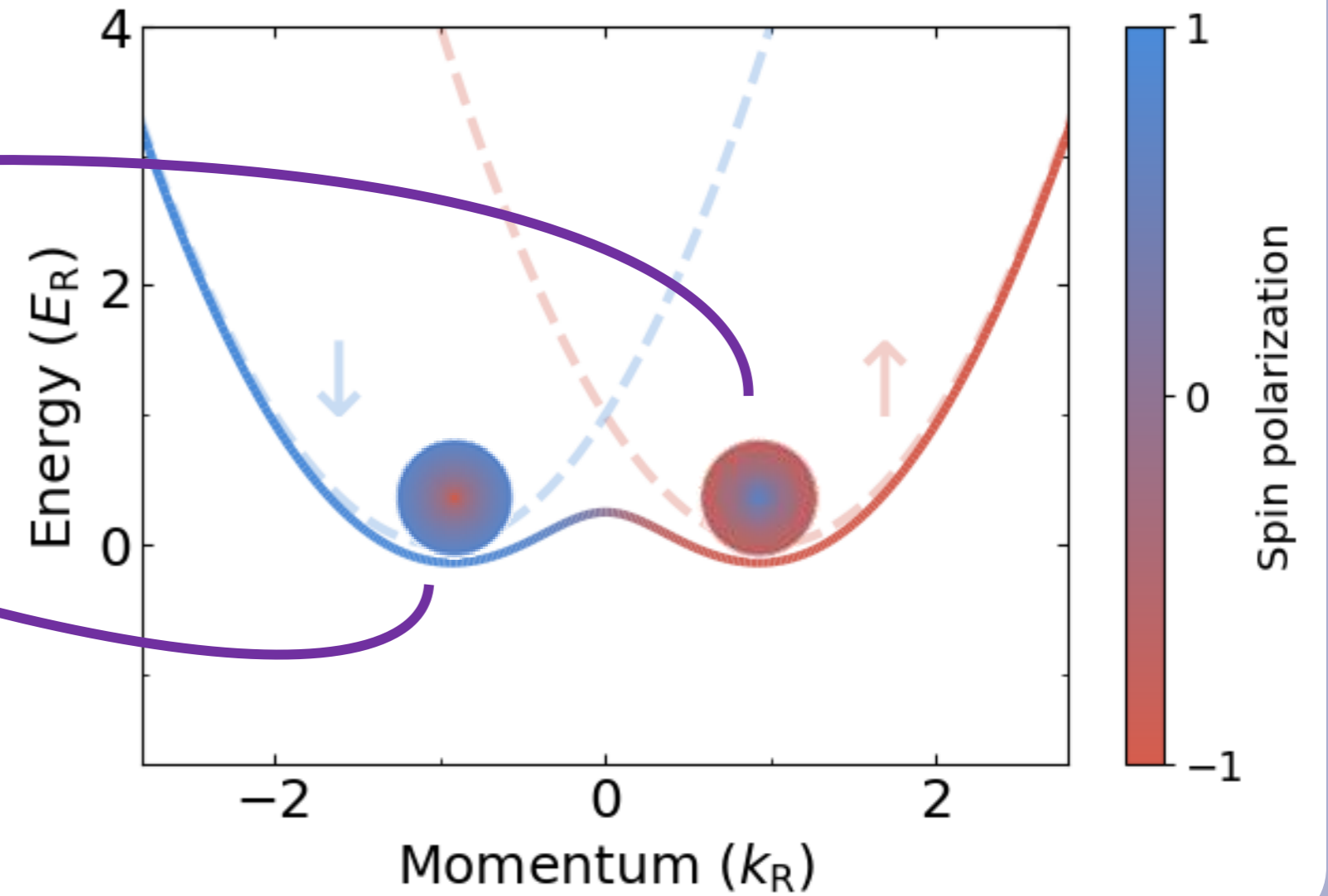
Stripe phase



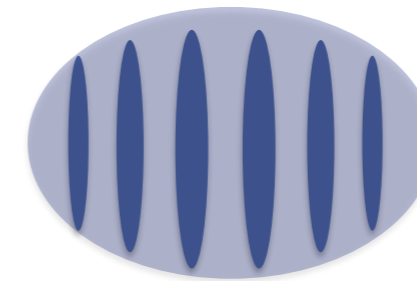
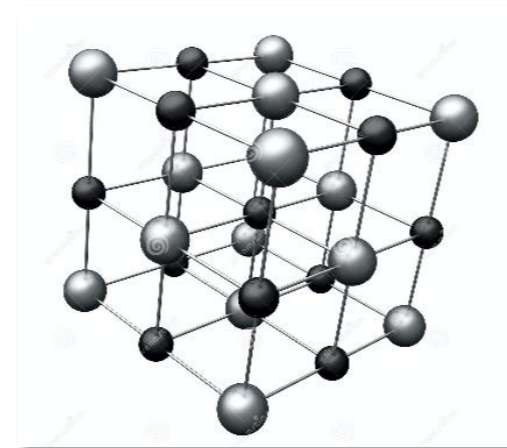
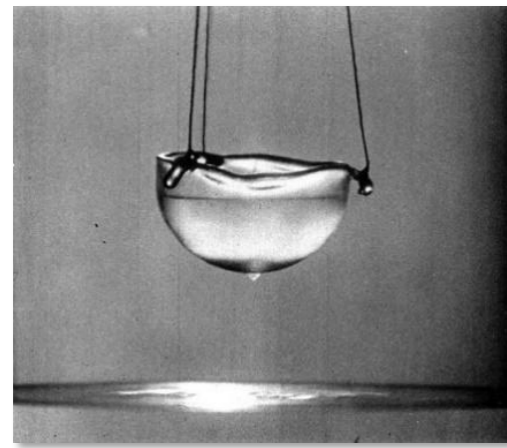
Plane-wave phase



Spin-orbit coupling



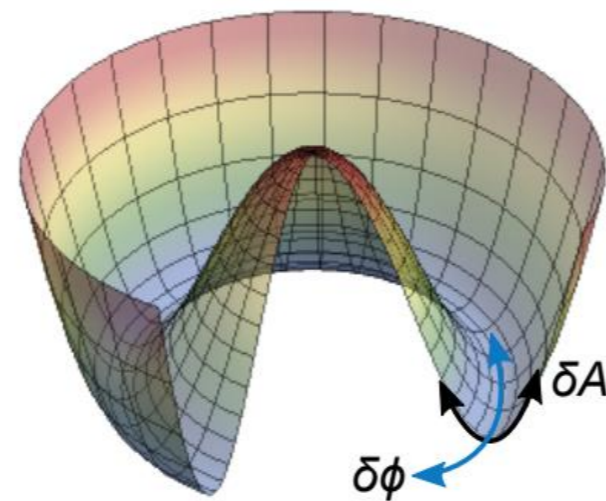
Superfluid Spontaneous U(1) symmetry breaking
Solid Spontaneous translational symmetry breaking
Supersolid both



Andreev and Lifshitz, *Sov. Phys. JETP* **29**, 1107 (1969)

Chester, *Phys. Rev. A* **2**, 256 (1970)

Leggett, *Phys. Rev. Lett* **25**, 1543 (1970)



two independent order parameters
➤ superfluid and crystal excitations

FEATURED IN PHYSICS

Absence of Supersolidity in Solid Helium in Porous Vycor Glass

[Duk Y. Kim](#) and [Moses H. W. Chan](#)*

Show more ▾

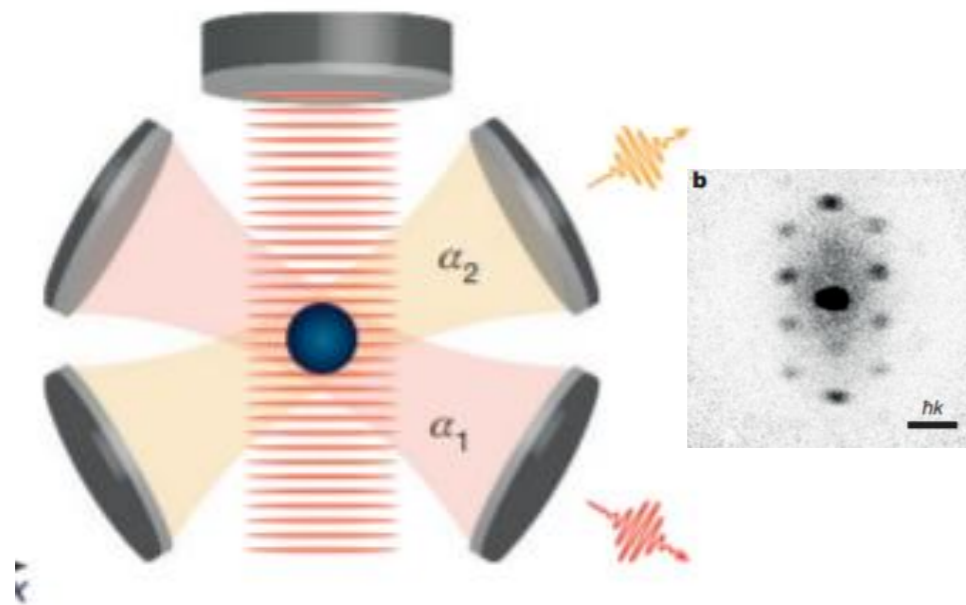
Phys. Rev. Lett. **109**, 155301 – Published 8 October, 2012

DOI: <https://doi.org/10.1103/PhysRevLett.109.155301>

crystalline structure
&
phase coherence

Balibar, *Nature* **464**, 176 (2010)

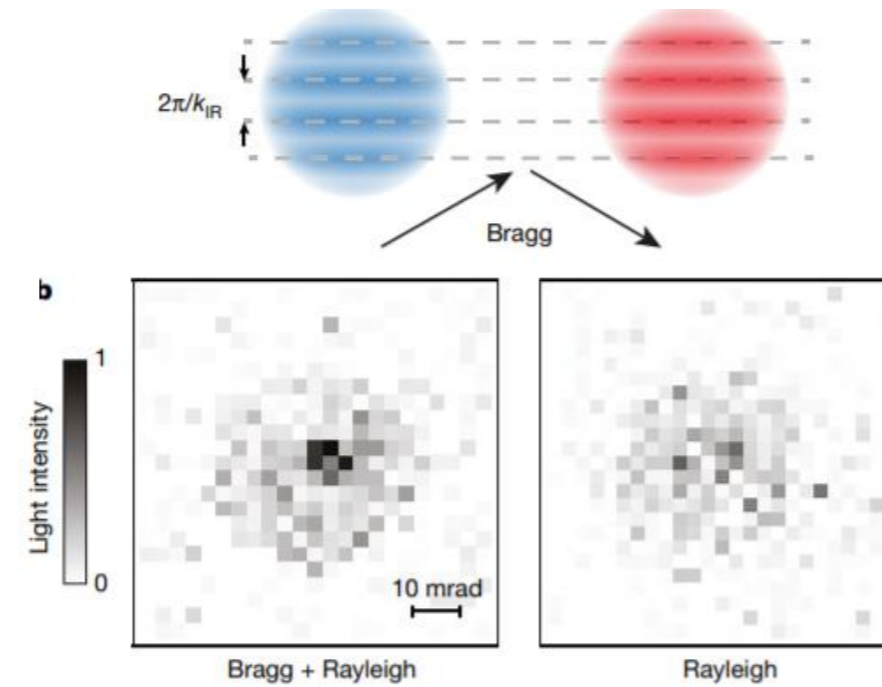
Cavity supersolids



Léonard et al., *Nature* **543** (2017)
Léonard et al., *Science* **358** (2017)

Period fixed by light wavelength
Rigid crystal

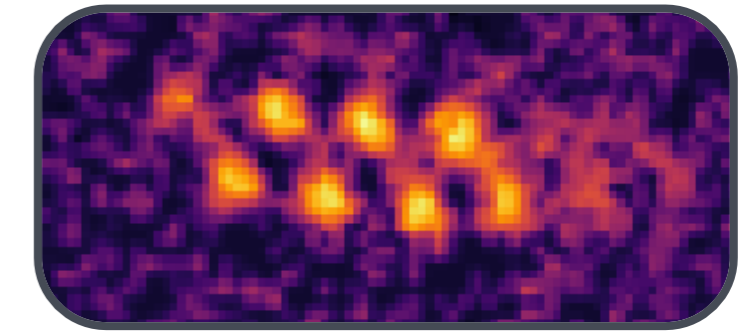
Spin-orbit coupled BECs



Li et al., *Nature* **543** (2017)
Putra et al., *PRL* **124** (2020)

Dynamics so far inaccessible
?

Dipolar supersolids

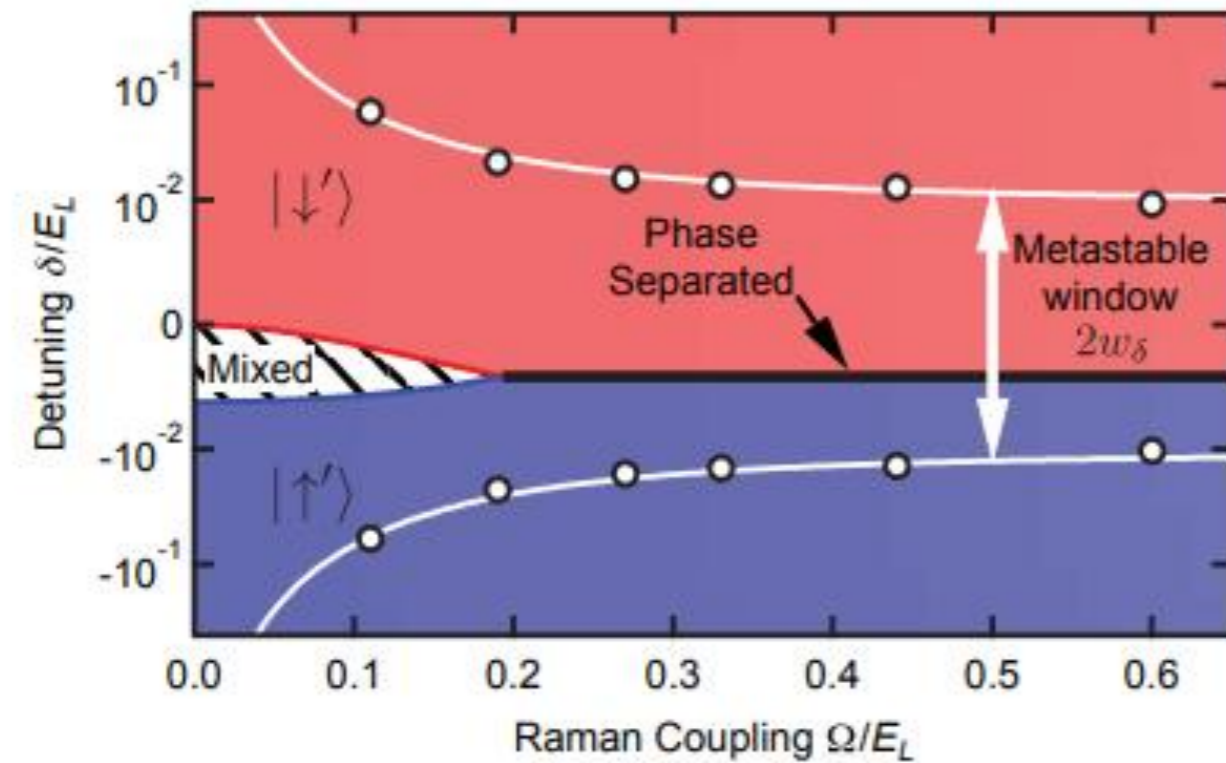


Norcia et al. *Nature* (2024)

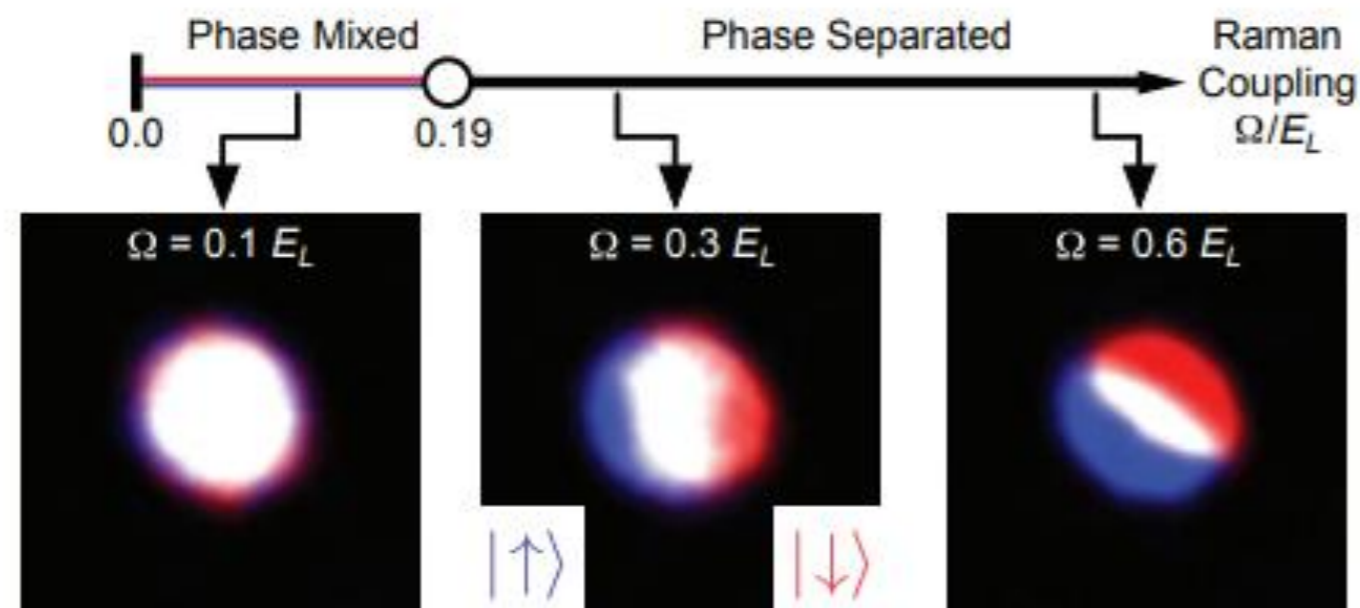
Tanzi et al., *PRL* **122** (2019)
Chomaz et al., *PRX* **9** (2019)
Böttcher et al., *PRX* **9** (2019)

...

Many excitation modes observed
Hosts phonon modes



c Miscible to immiscible transition



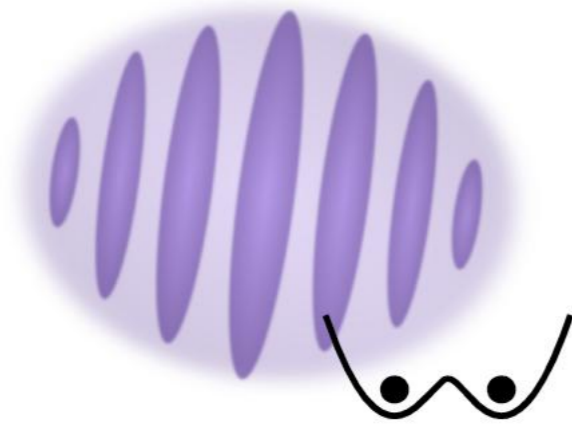
Lin et al. *Nature* **471** (2011)

Problems:

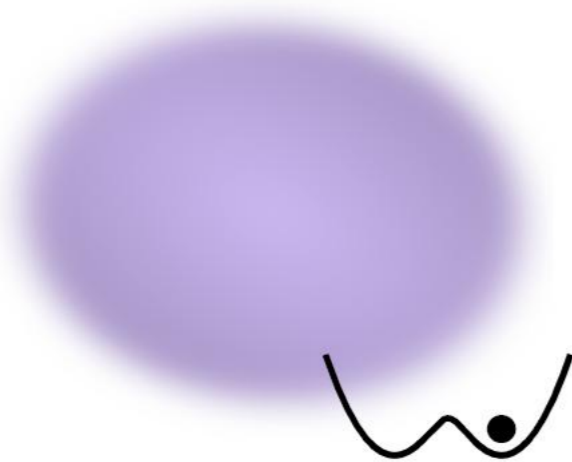
- unstable stripe phase
- phase separation
- small modulation contrast $C \sim \Omega$

Matter wave interference

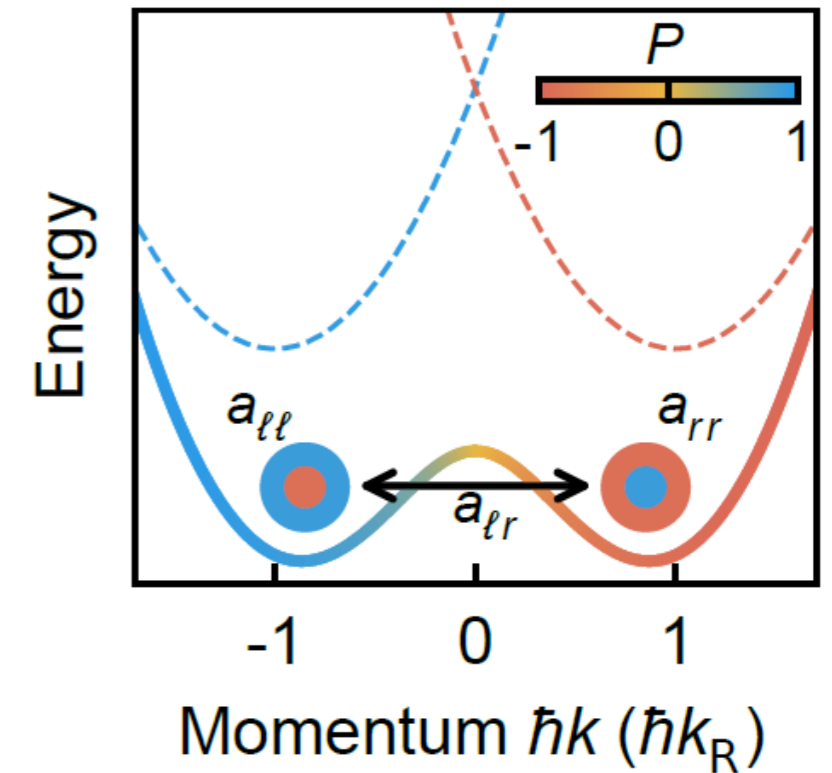
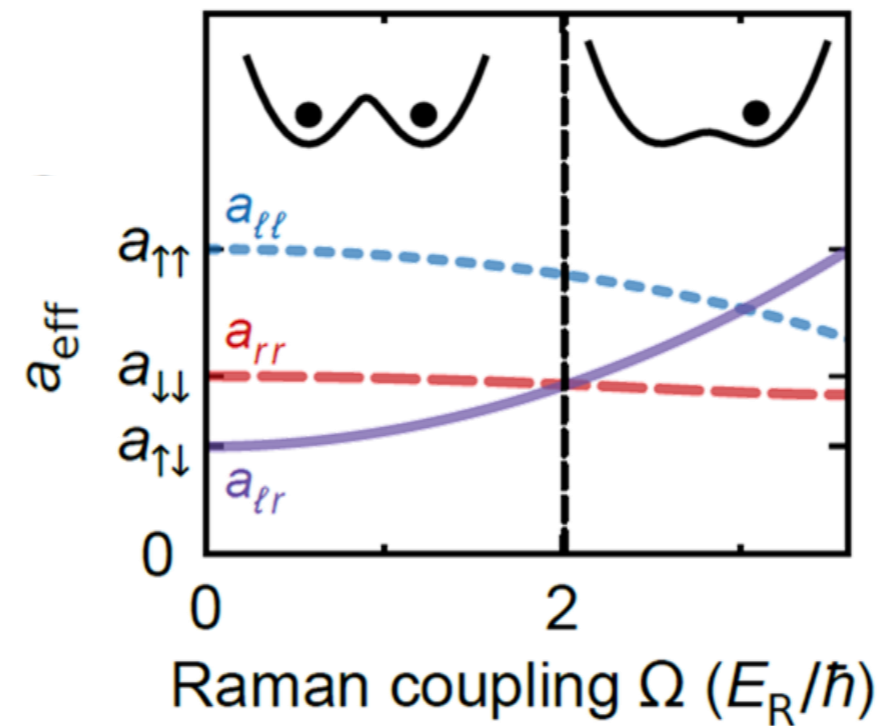
Supersolid stripe phase



Plane-wave phase

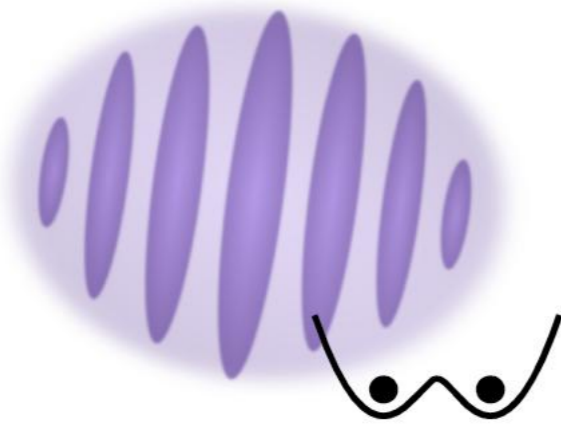


Phase diagram determined by effective interactions

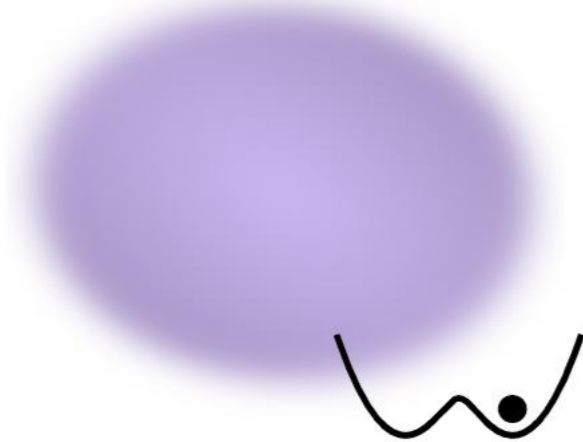


Matter wave interference

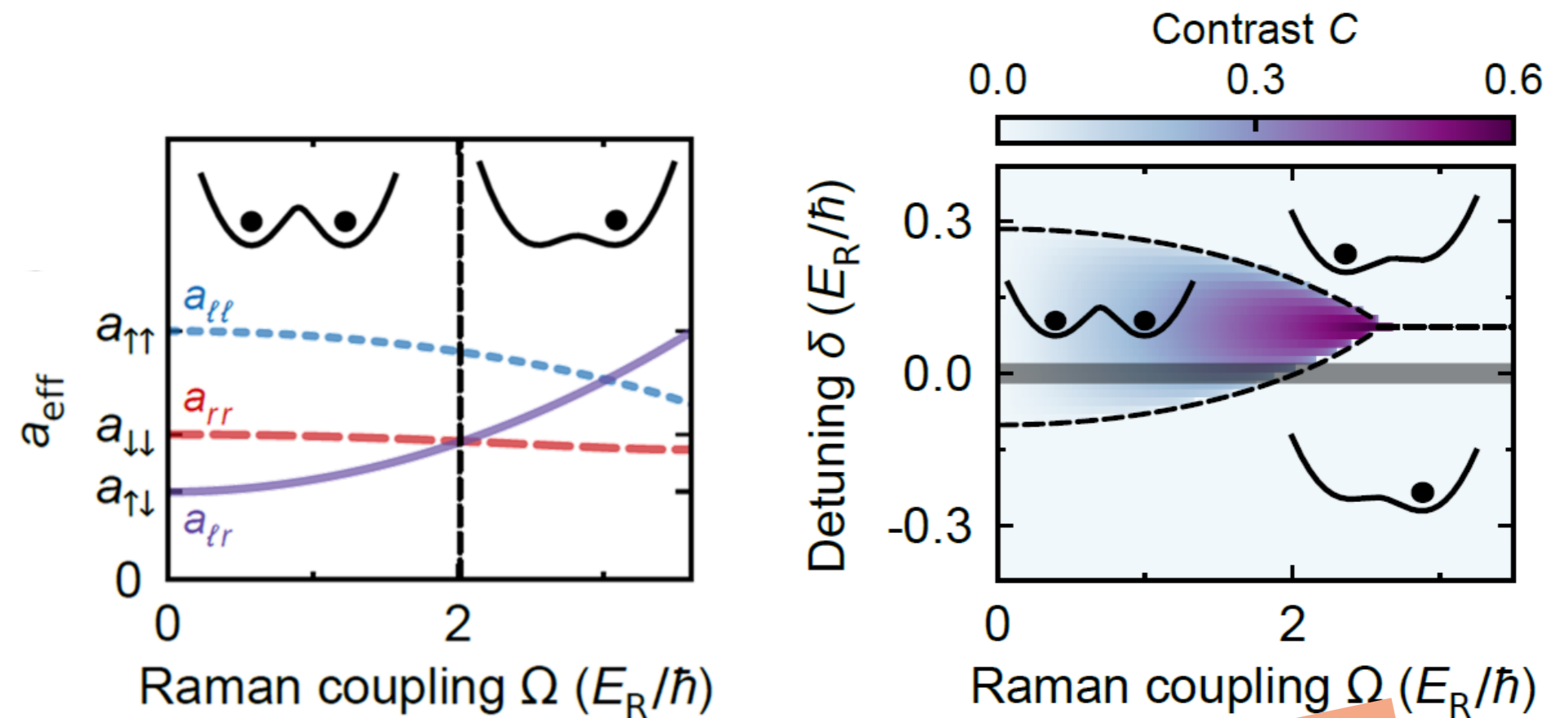
Supersolid stripe phase



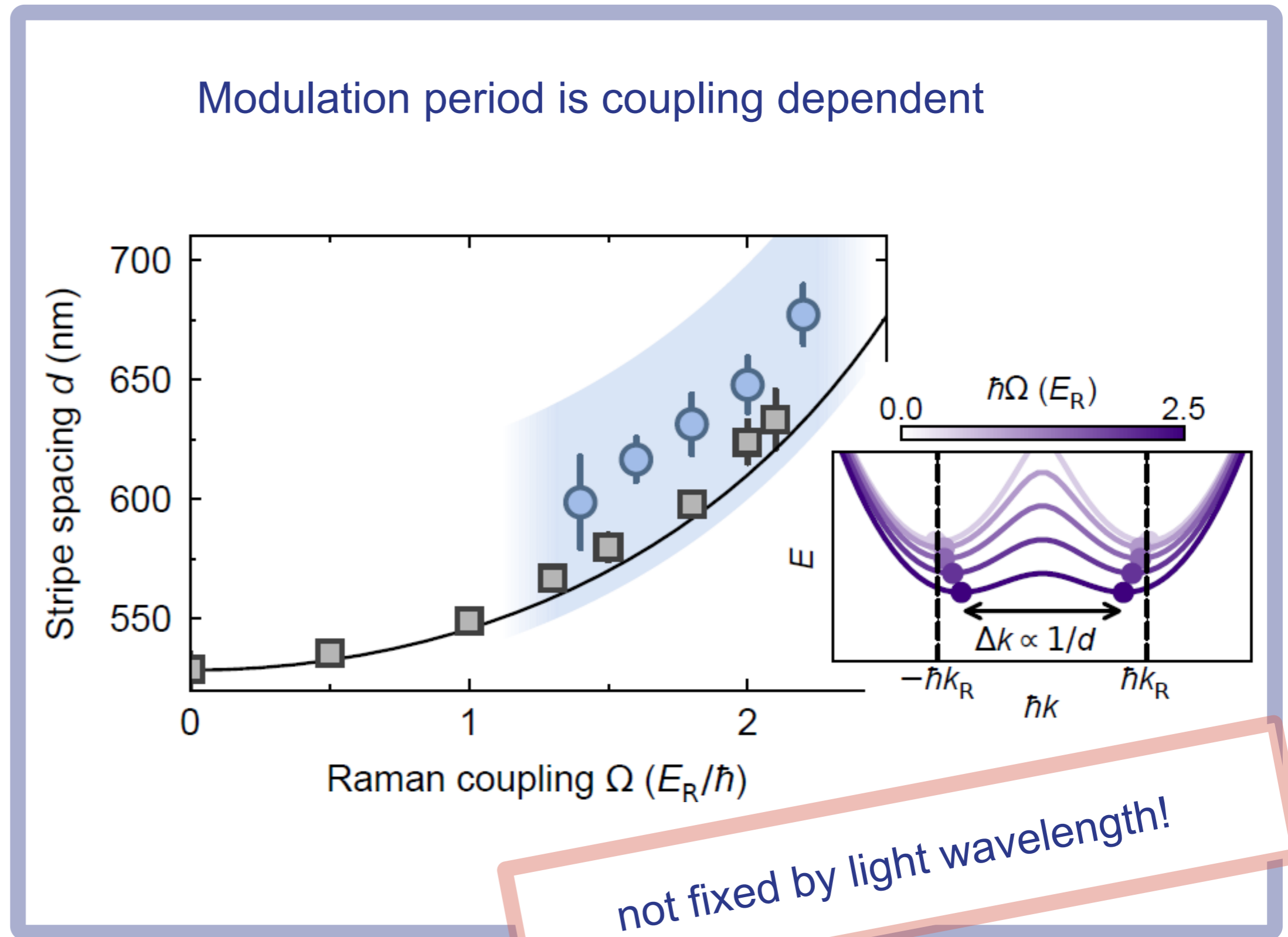
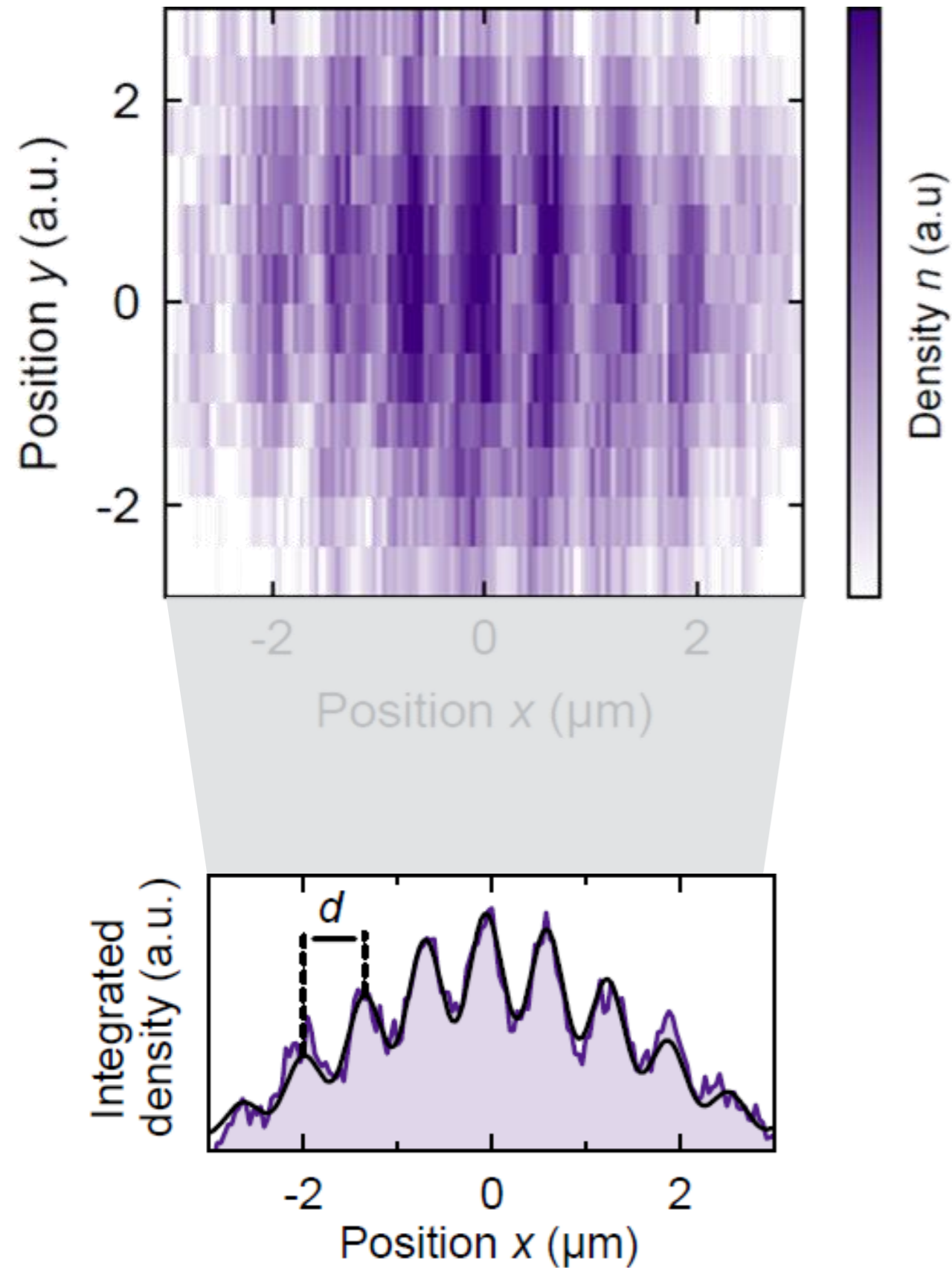
Plane-wave phase



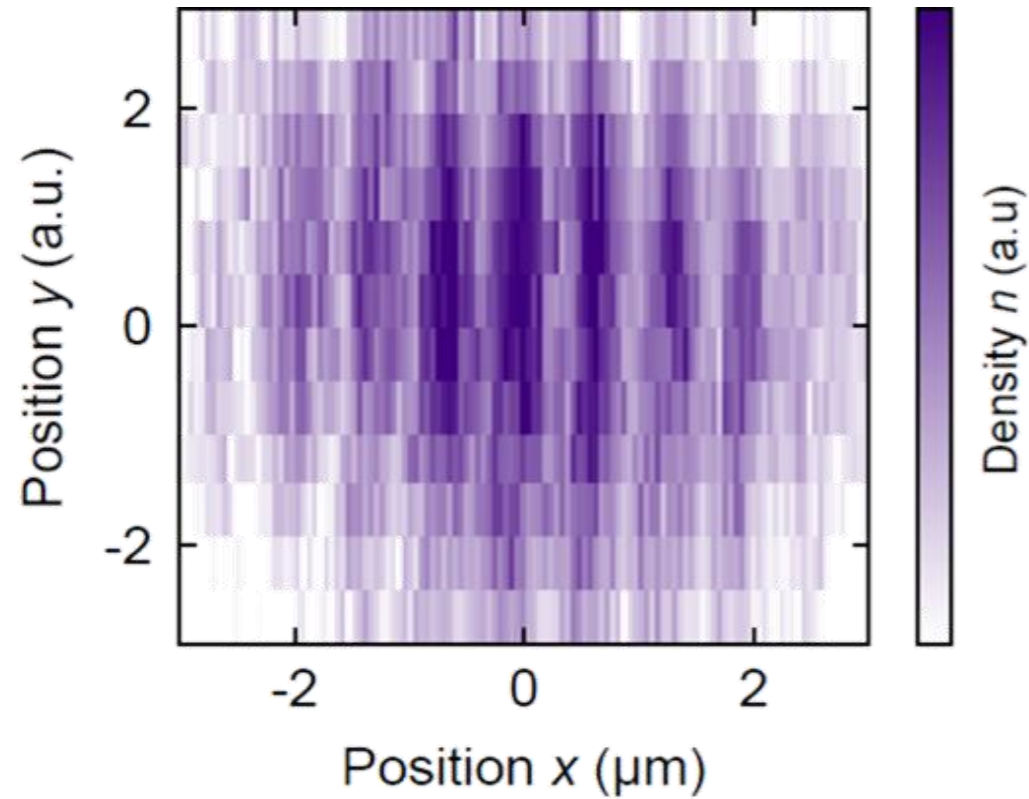
Phase diagram determined by effective interactions



Stable stripe phase with high contrast

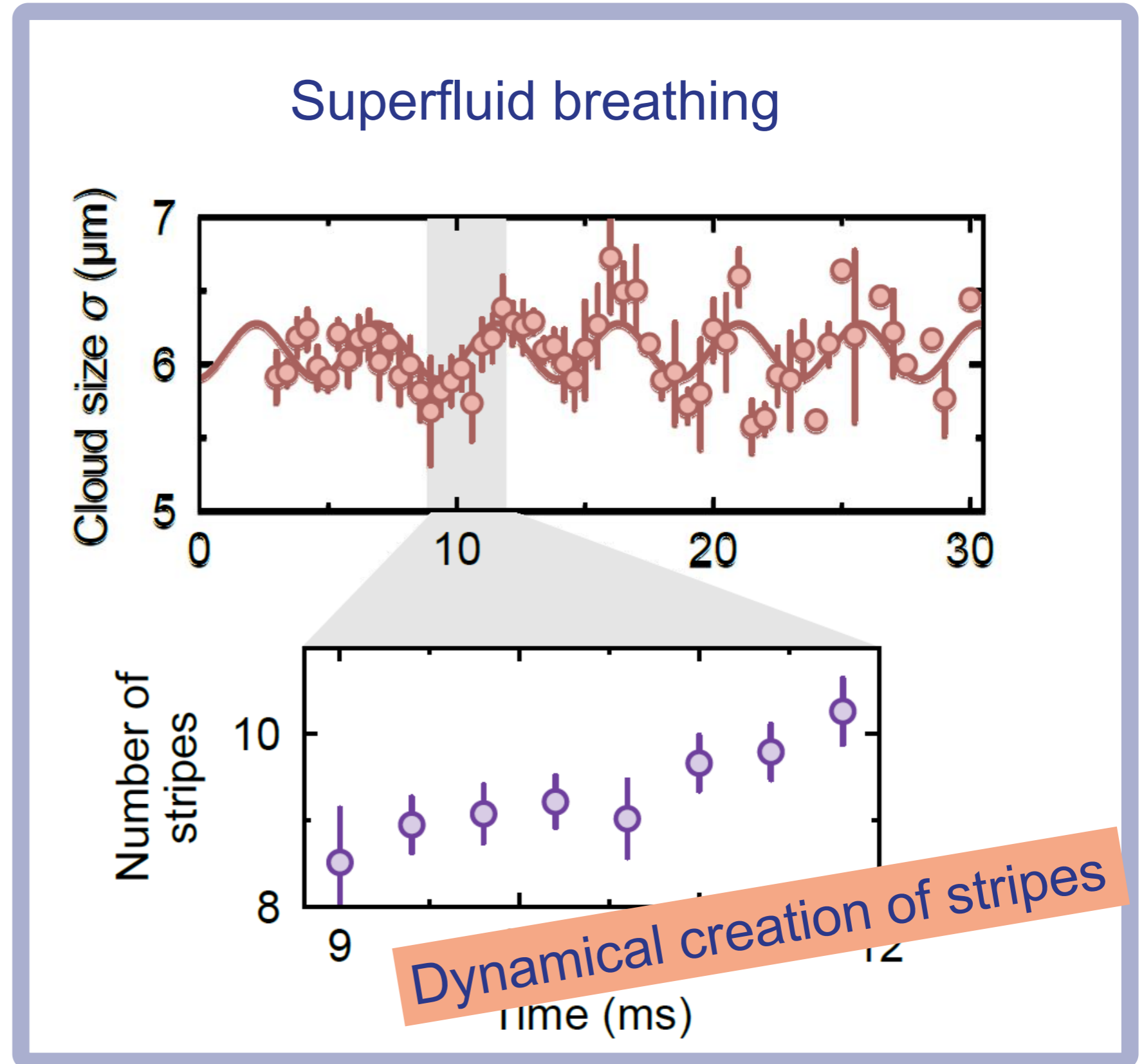


not fixed by light wavelength!

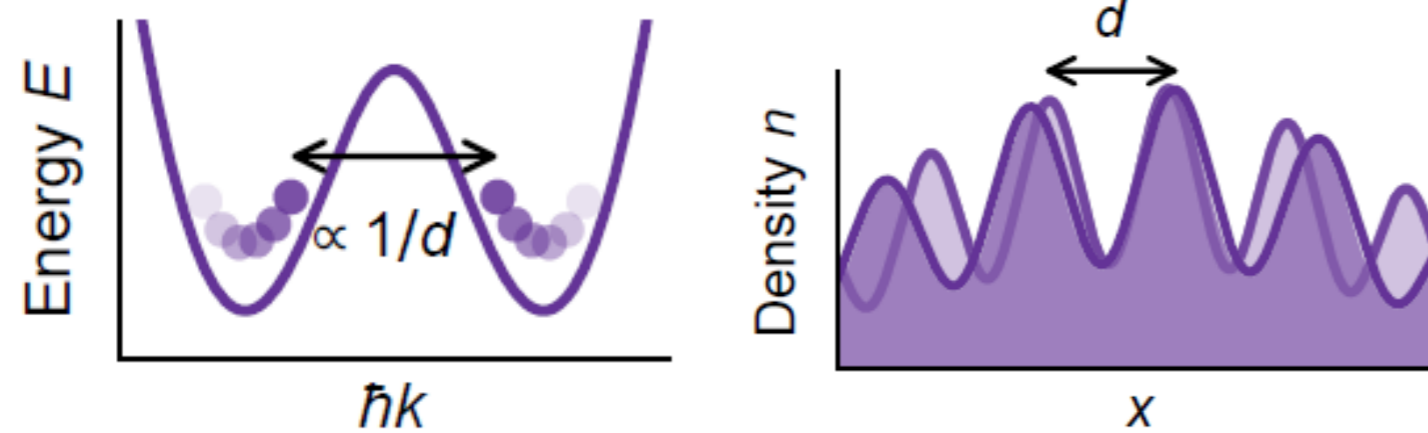


$$\frac{\omega_{\text{breathing}}}{\omega_{\text{dipole}}} = 1.5(1)$$

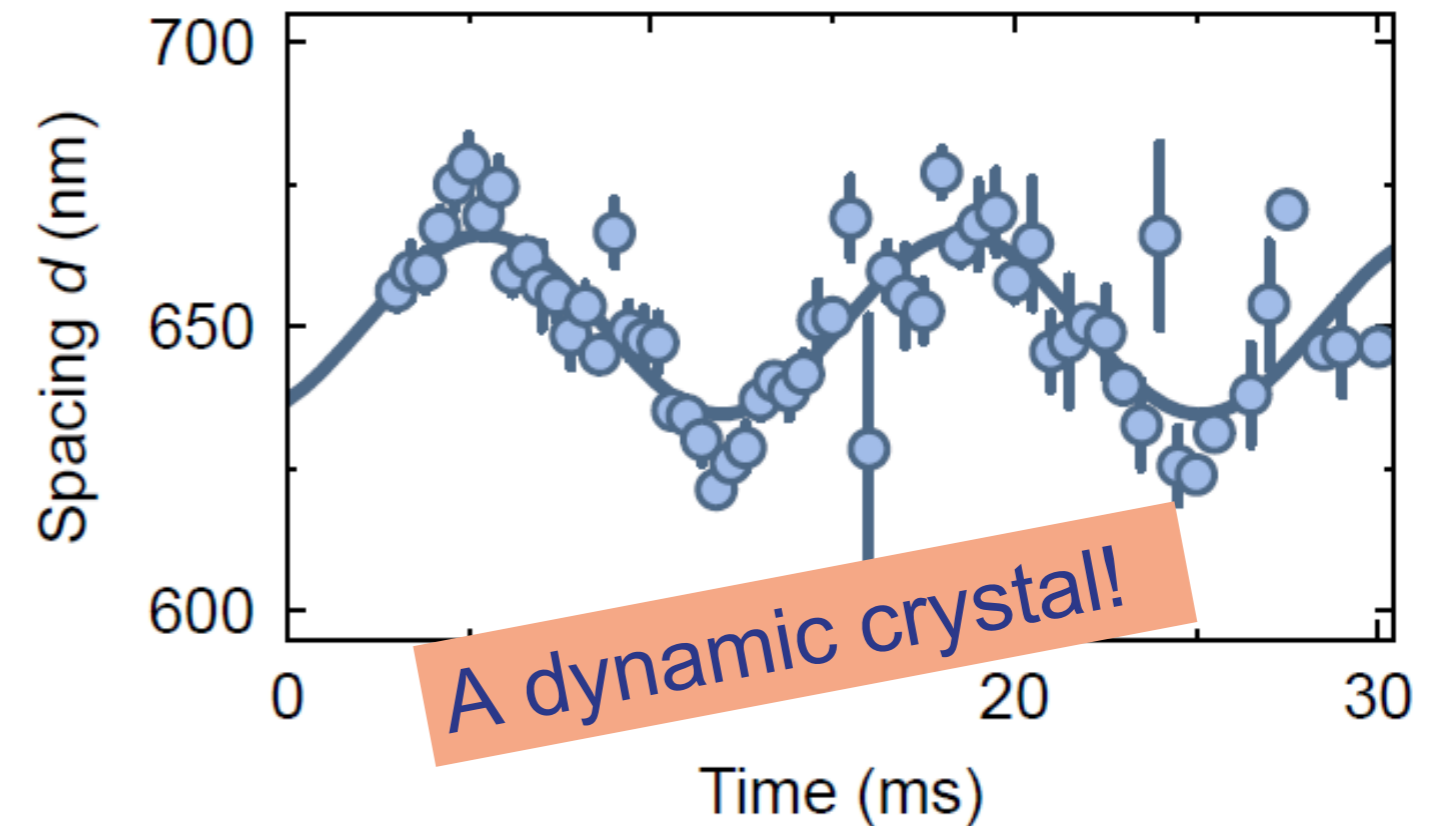
in good agreement with conventional BEC prediction



Stripe compression mode



Oscillation of the modulation period



What happens at the transition?

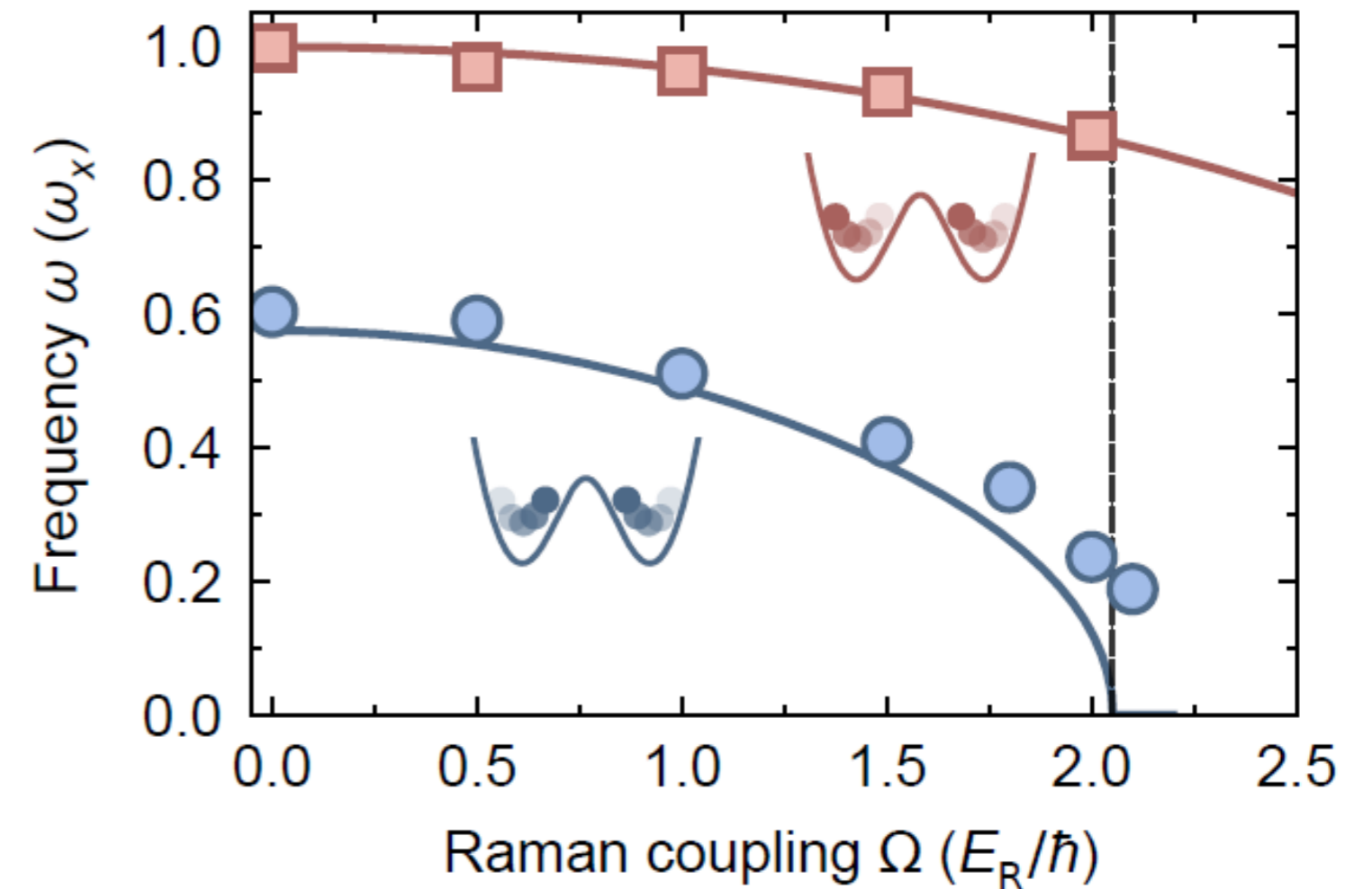
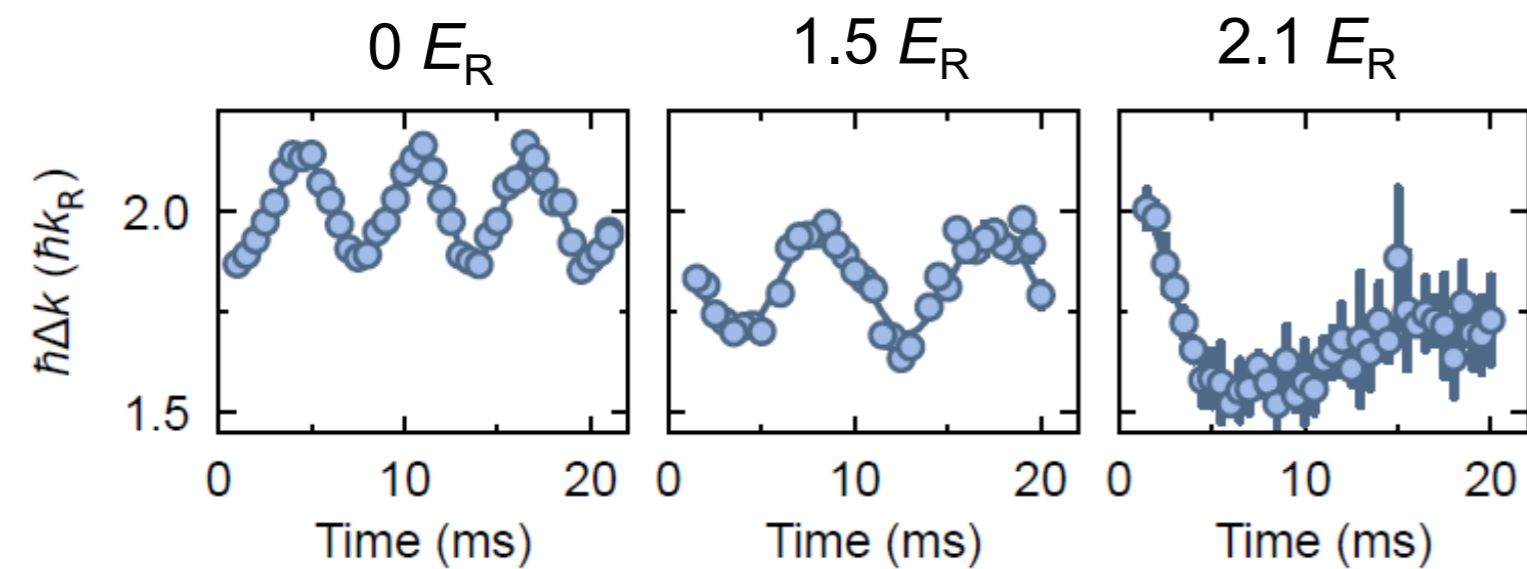
Theory of collective modes in SOC supersolids:

- Geier et al., *PRL* **130** (2023)
- Geier et al. *PRL* **127** (2021)
- Martone and Stringari *SciPost Phys.* **11** (2021)
- Chen et al. *PRA* **98** (2017)

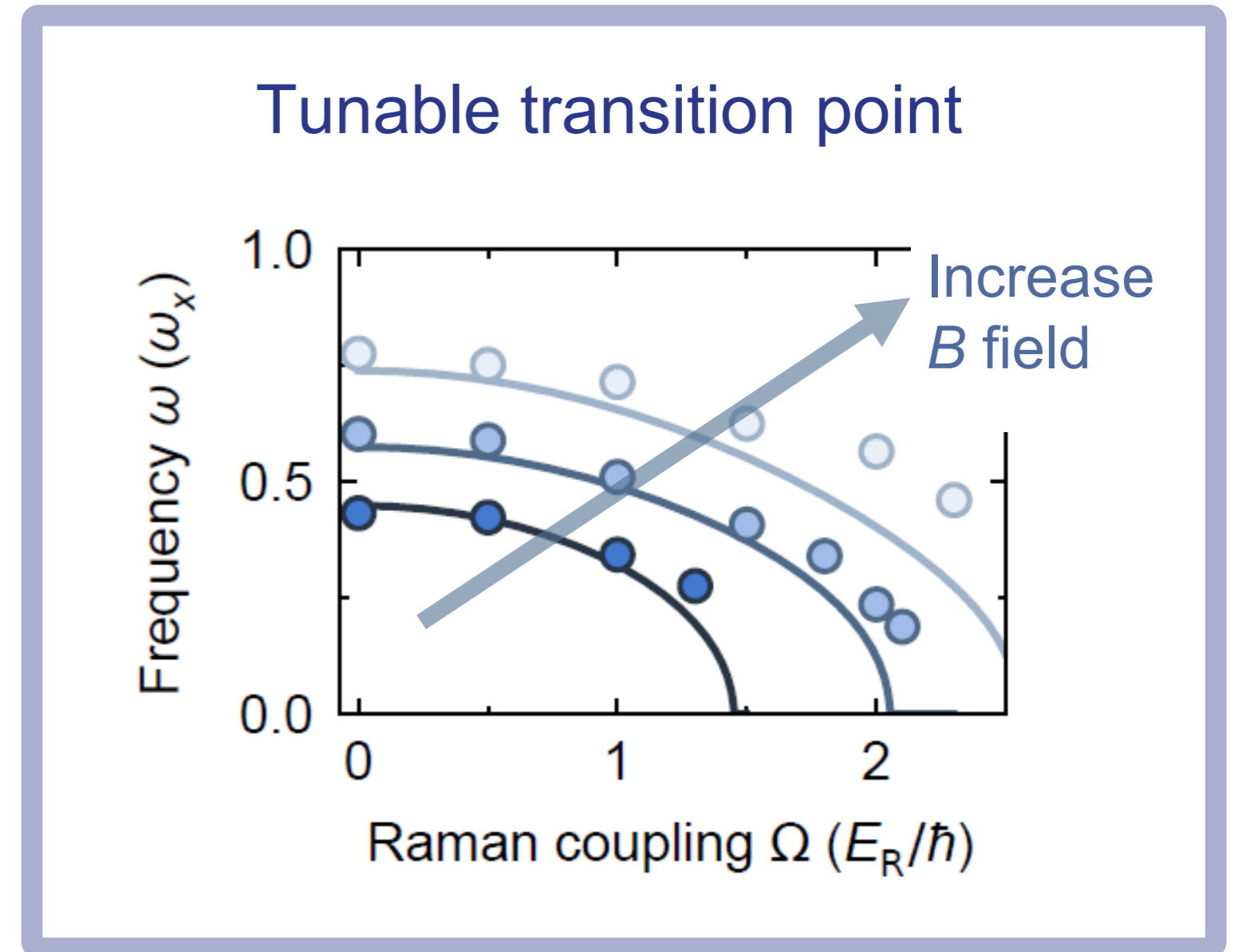
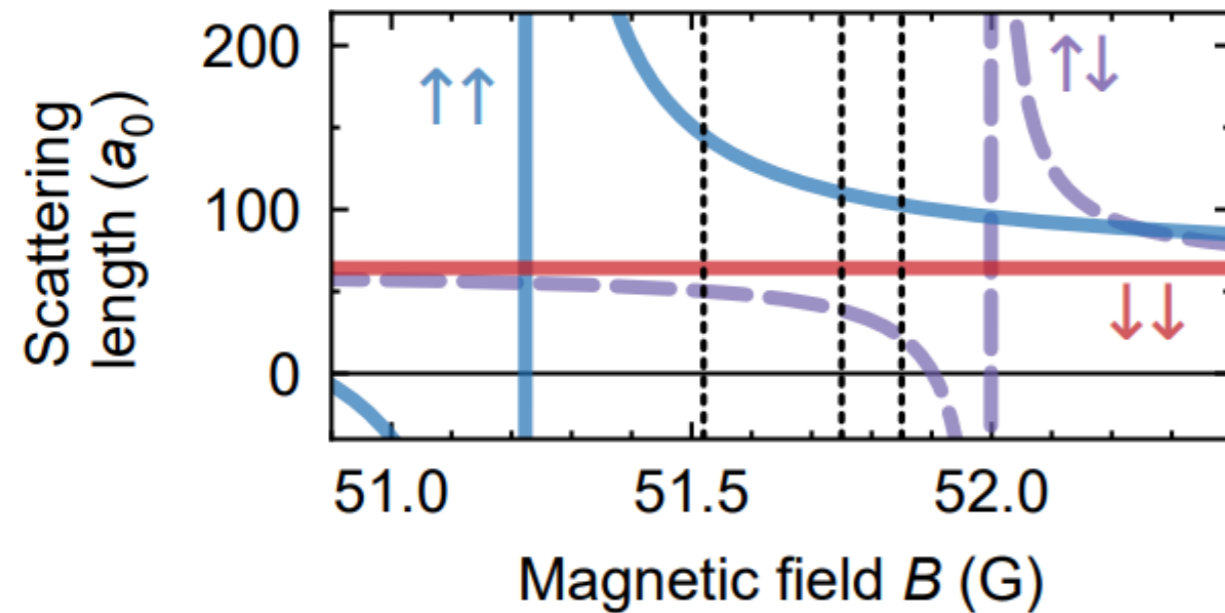
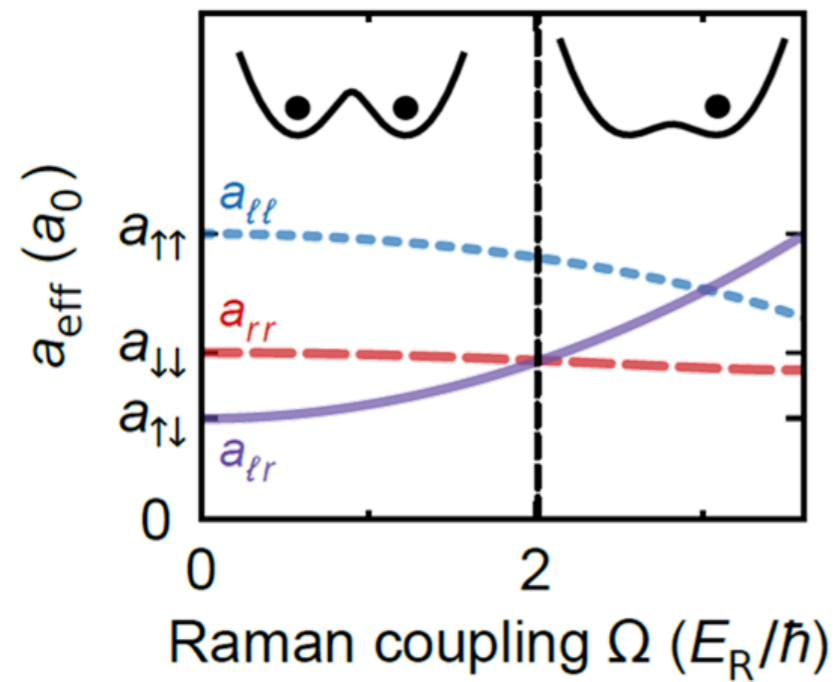
...

Compression mode softens at phase transition

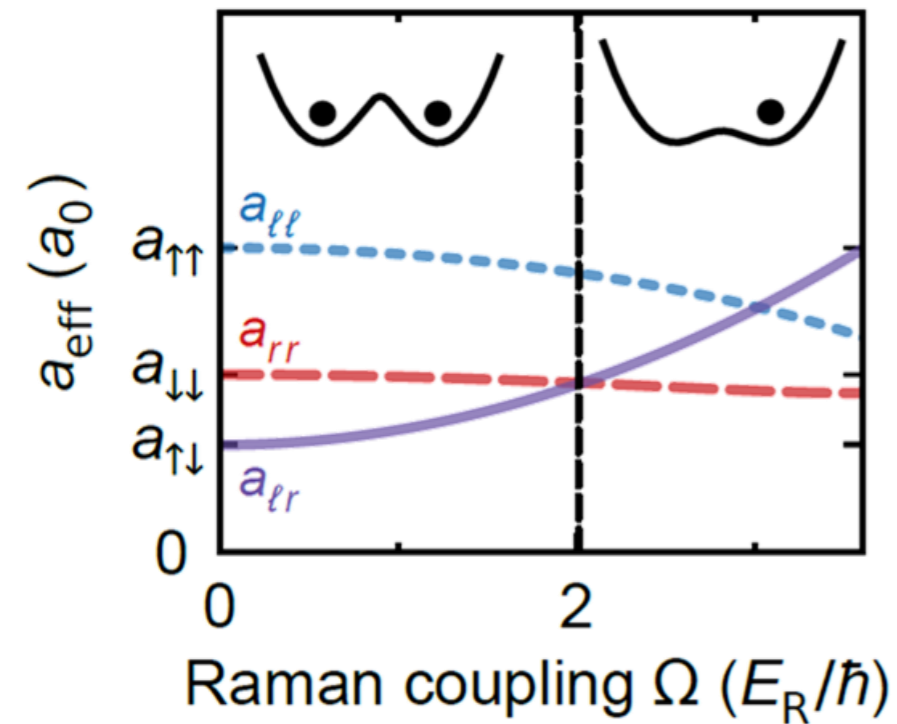
Frequency softening



Tuning interactions

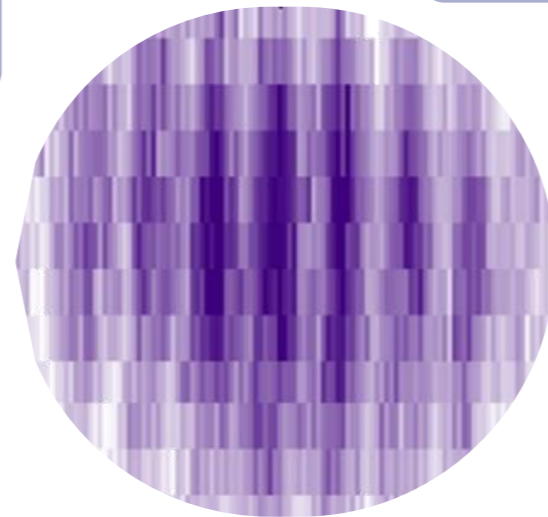
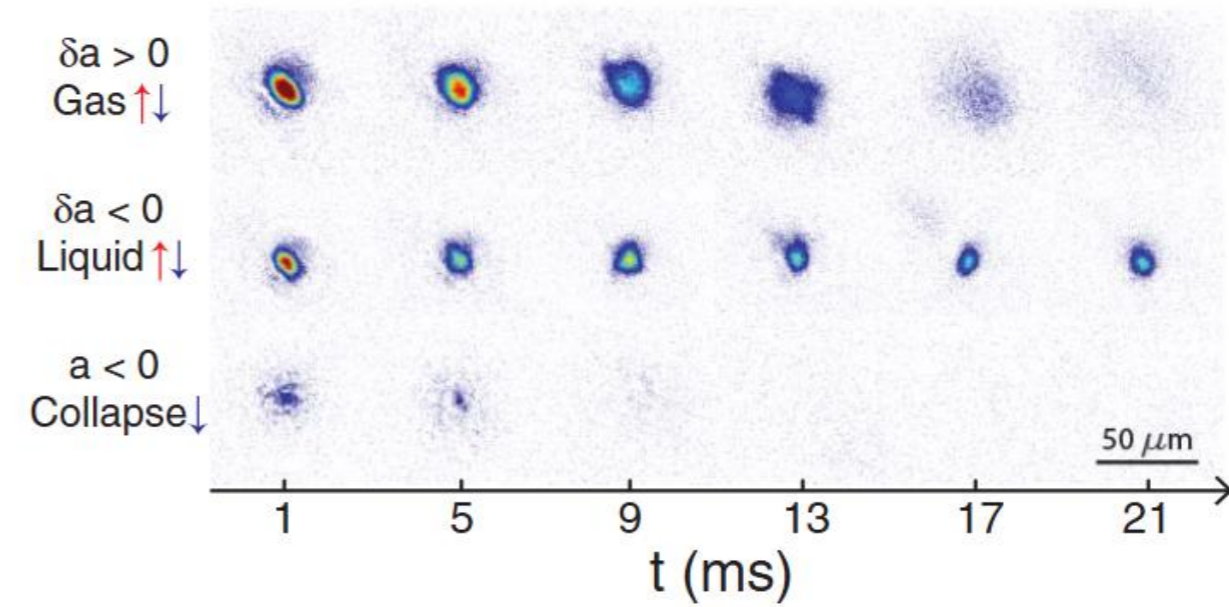


Spin-Orbit Coupling



+

Beyond-mean field effects



Self-bound supersolid

I. Bosonic mixtures with tunable interactions

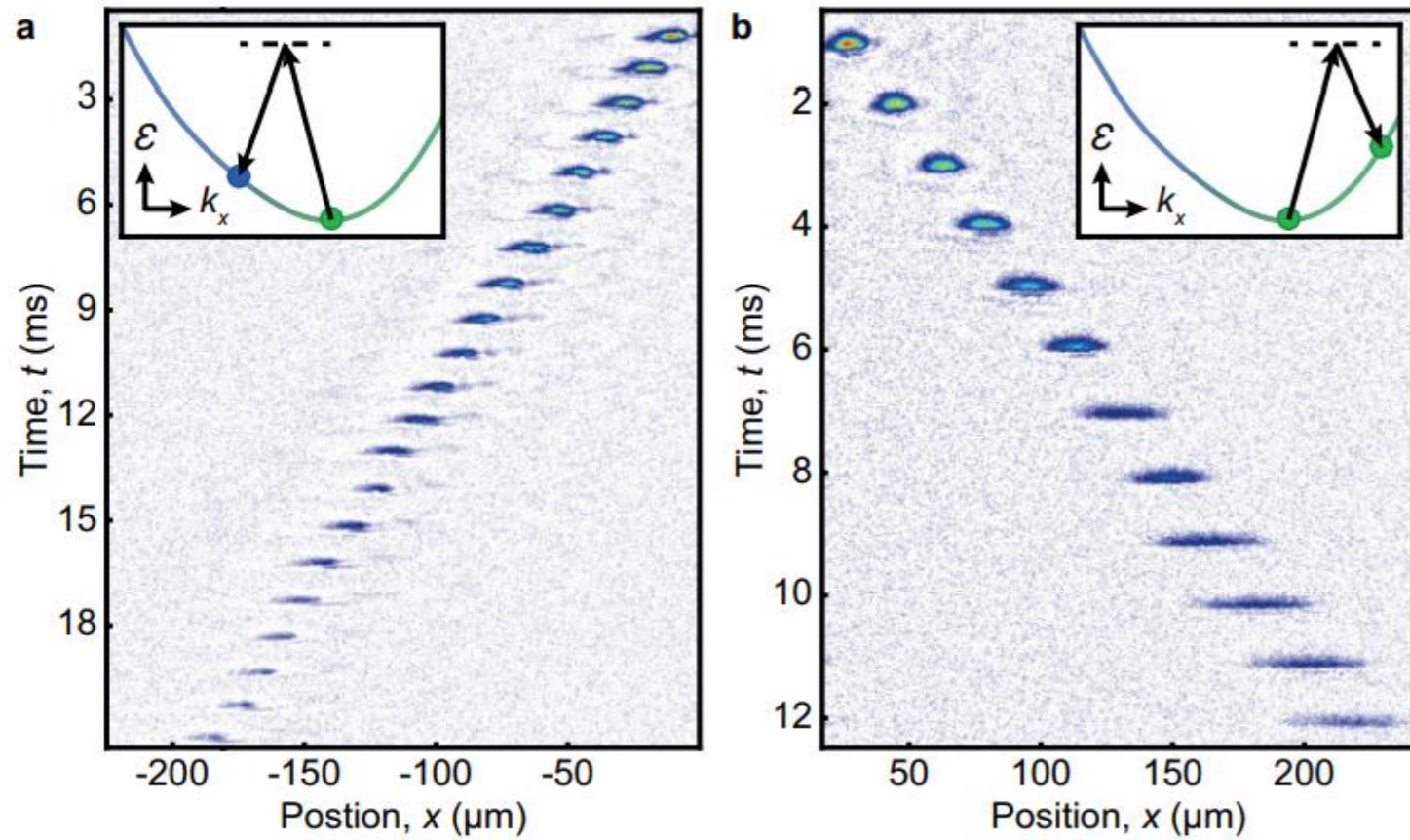
Quantum liquid droplets

II. Raman-coupled mixtures

Supersolids

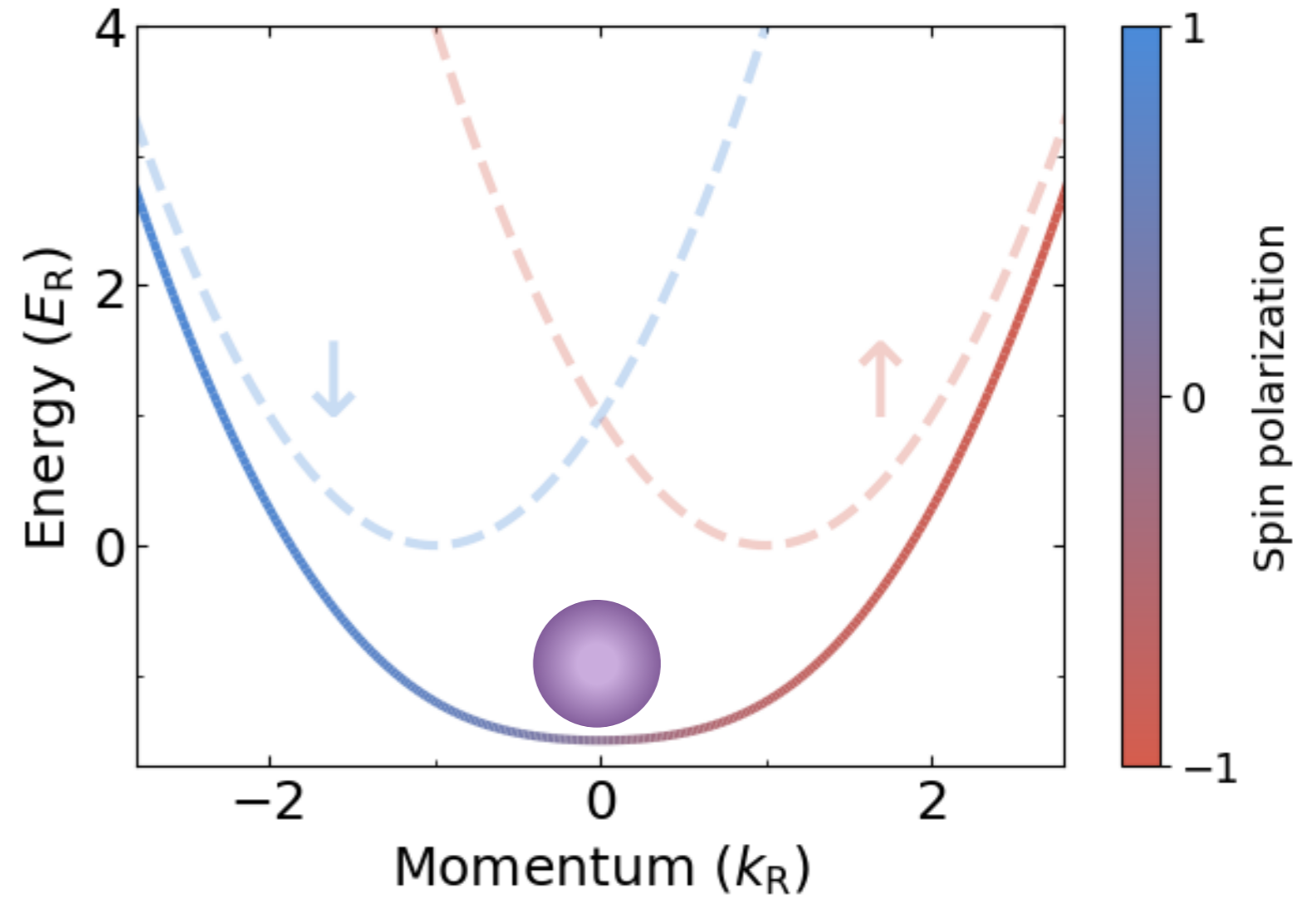
Chiral solitons

Chiral solitons



Frölian et al., *Nature* **608**, 293 (2022)

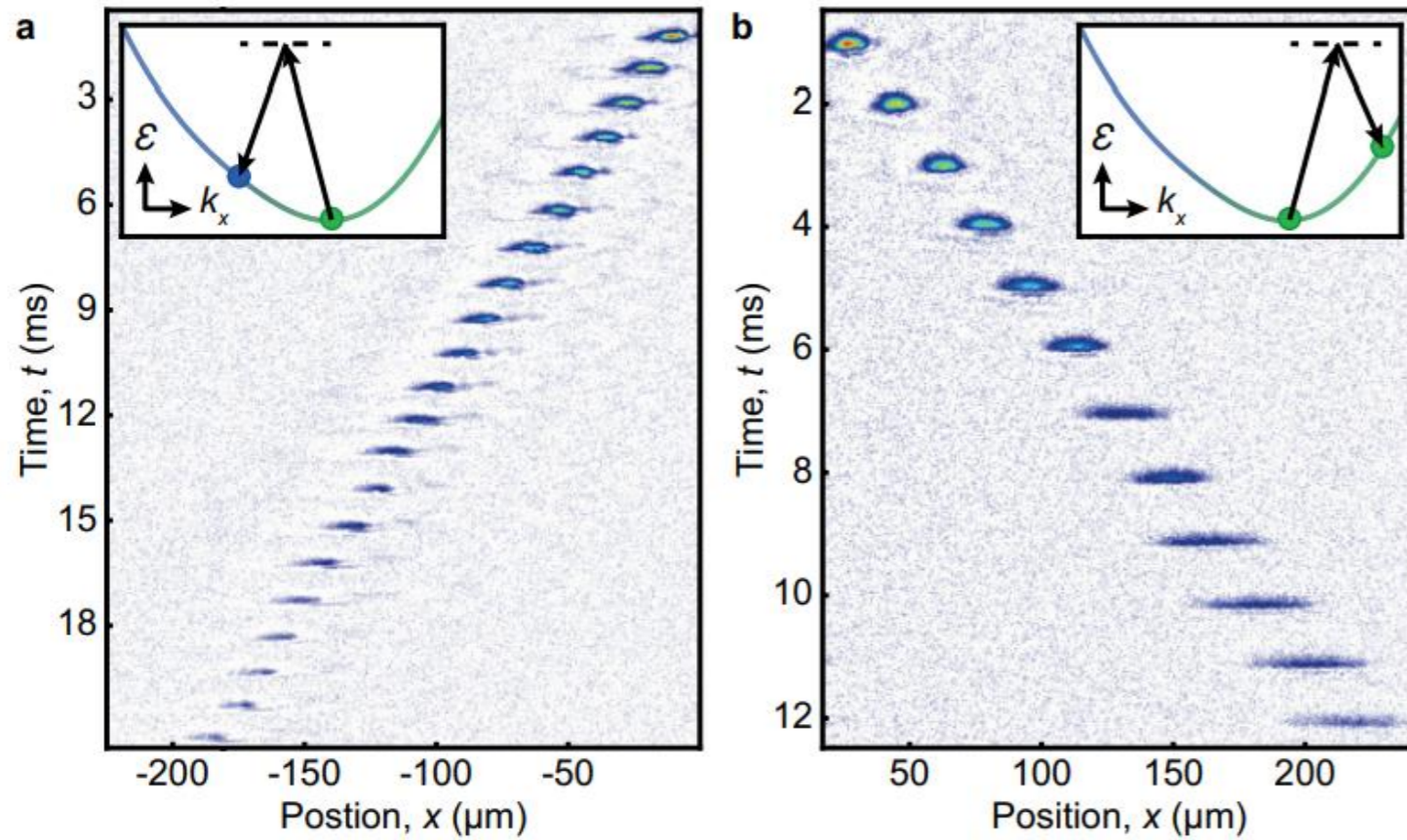
Engineered dispersion relation



$a_{\uparrow\uparrow}/a_0$	-4.9
$a_{\downarrow\downarrow}/a_0$	24.6
$a_{\uparrow\downarrow}/a_0$	-13.8

Simulation of a topological gauge theory

Chiral solitons



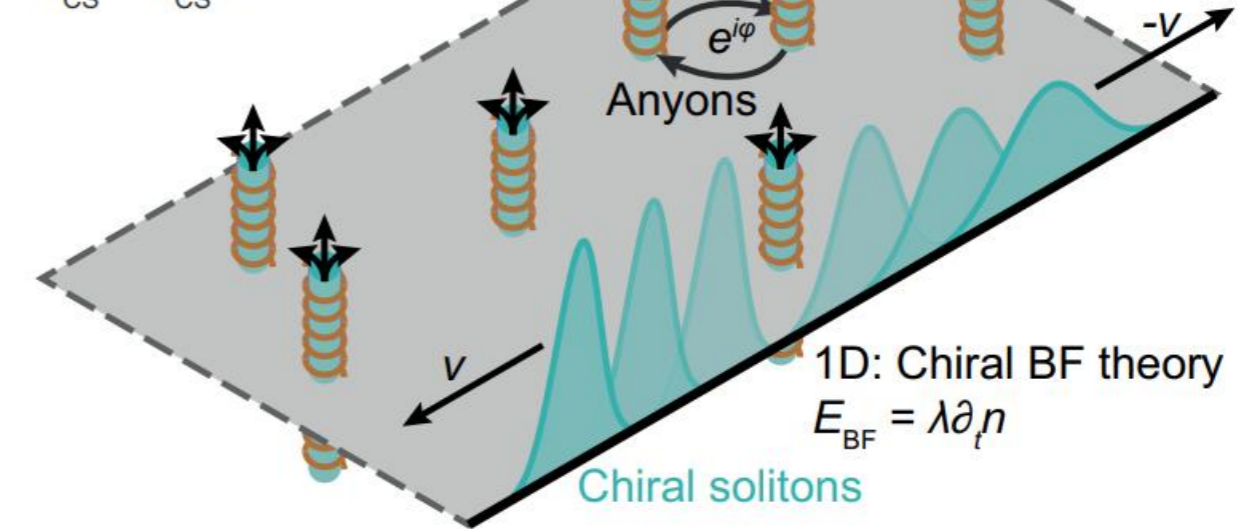
Frölian et al., *Nature* **608**, 293 (2022)



Chiral BF theory

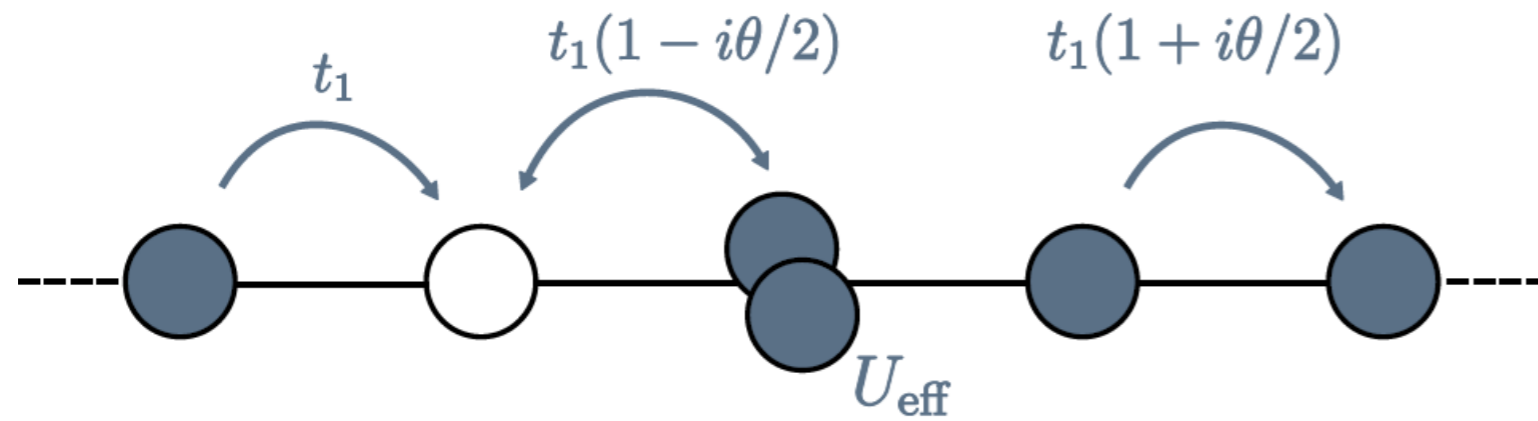
2D: Chern-Simons theory

$$B_{CS} = \kappa_{CS} n$$



$a_{\uparrow\uparrow}/a_0$	-4.9
$a_{\downarrow\downarrow}/a_0$	24.6
$a_{\uparrow\downarrow}/a_0$	-13.8

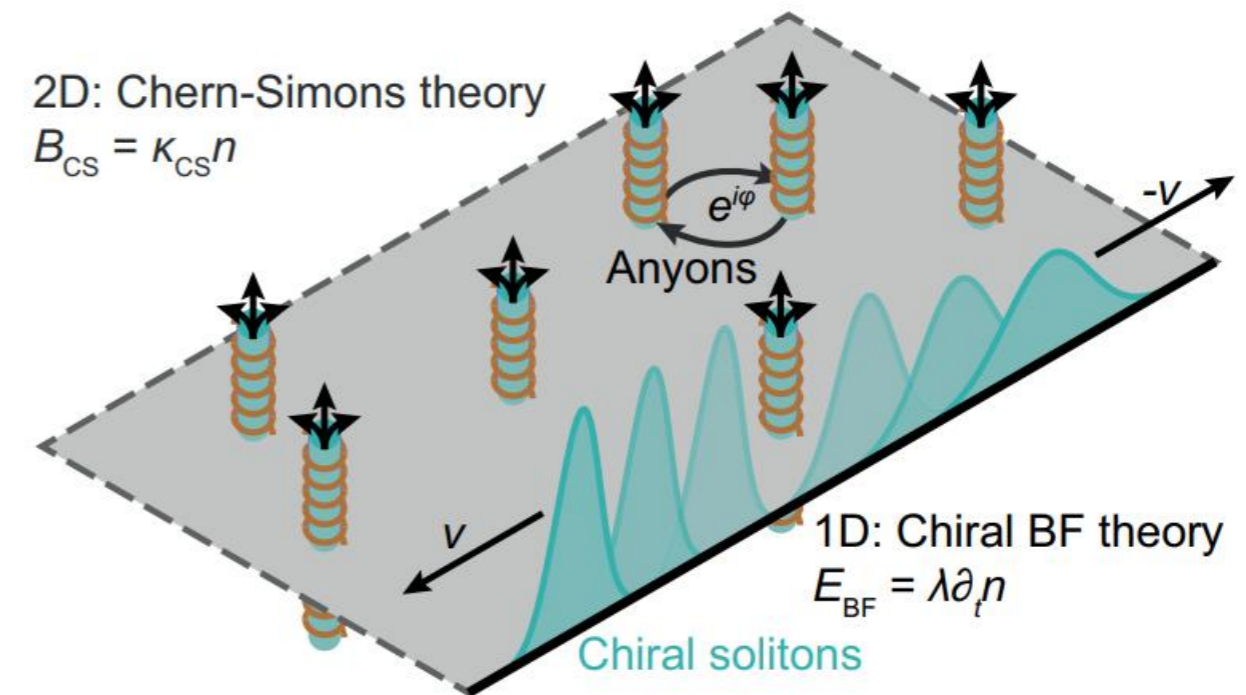
Linear anyons



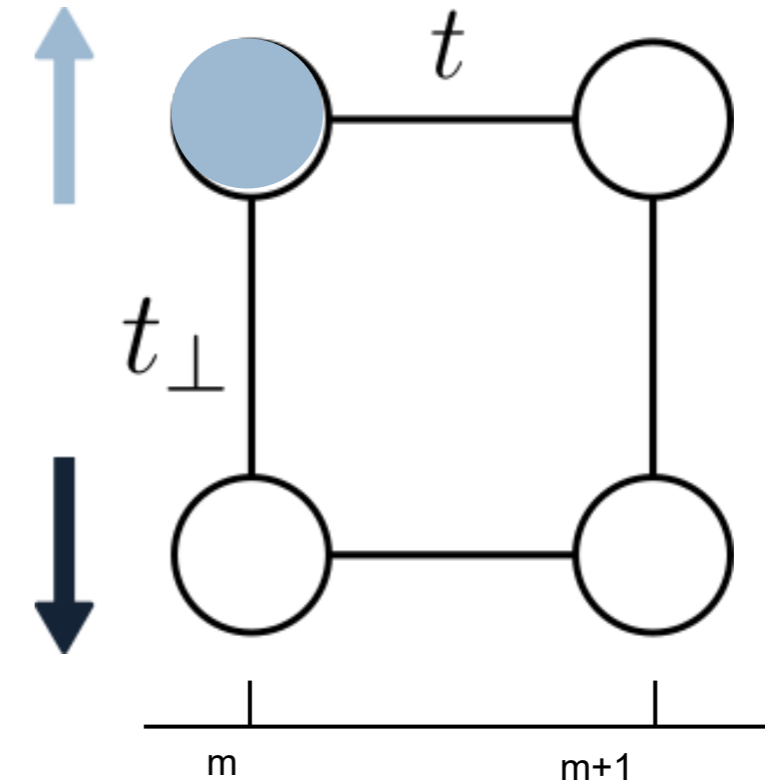
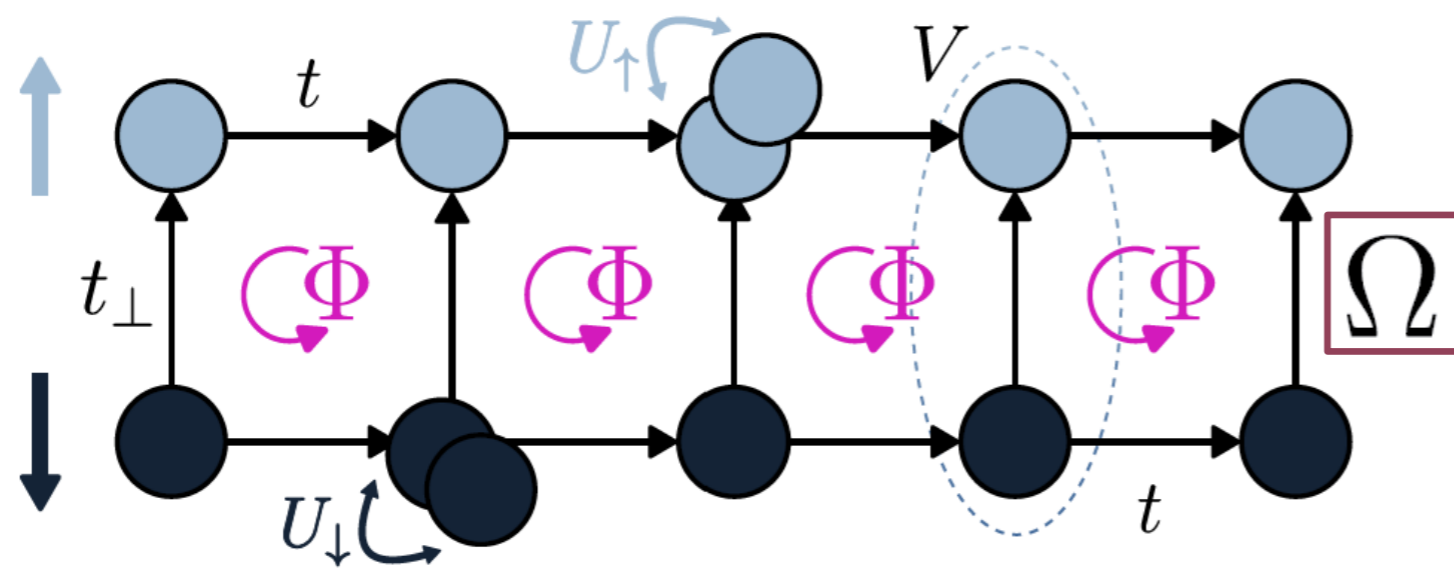
$$H = -J \sum_j \left(b_j^\dagger e^{-in_j\theta} b_{j-1} + \text{h.c.} \right) + \frac{U}{2} \sum_j n_j (n_j - 1)$$

Cabedo et al. *In preparation*

Chiral BF theory



Synthetic dimensions



N of flux quanta

$$\gamma = \sum \phi = 2k_R/d$$

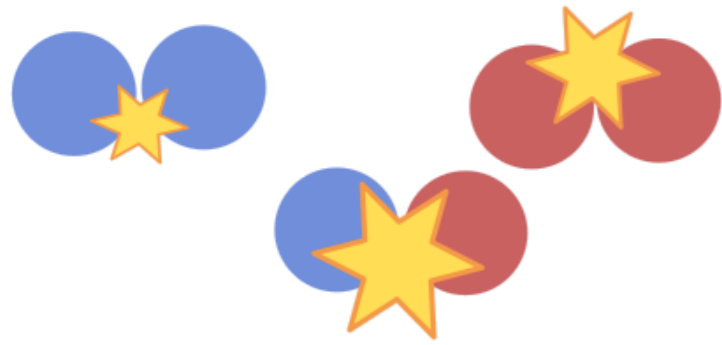
$$\phi_{m,\uparrow \rightarrow m+1,\uparrow} = 0$$

$$\phi_{m+1,\uparrow \rightarrow m+1,\downarrow} = -2k_R(m+1)d$$

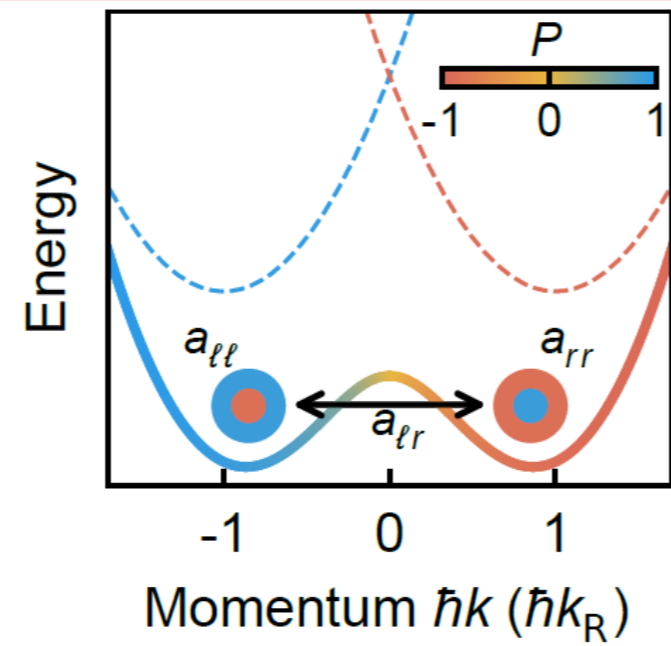
$$\phi_{m+1,\downarrow \rightarrow m,\downarrow} = 0$$

$$\phi_{m,\downarrow \rightarrow m,\uparrow} = 2k_Rmd$$

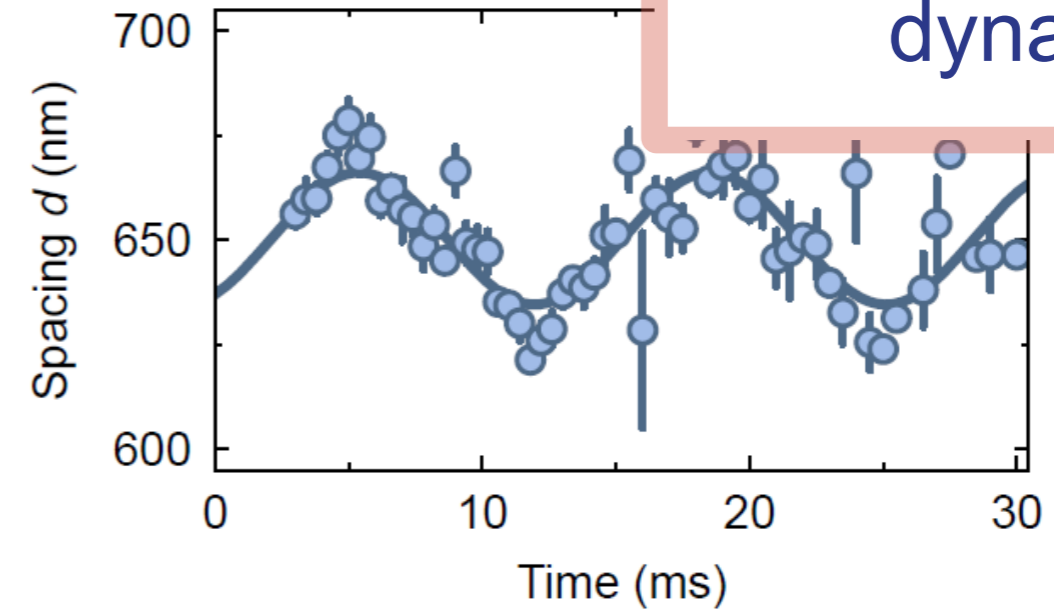
Tunable interactions



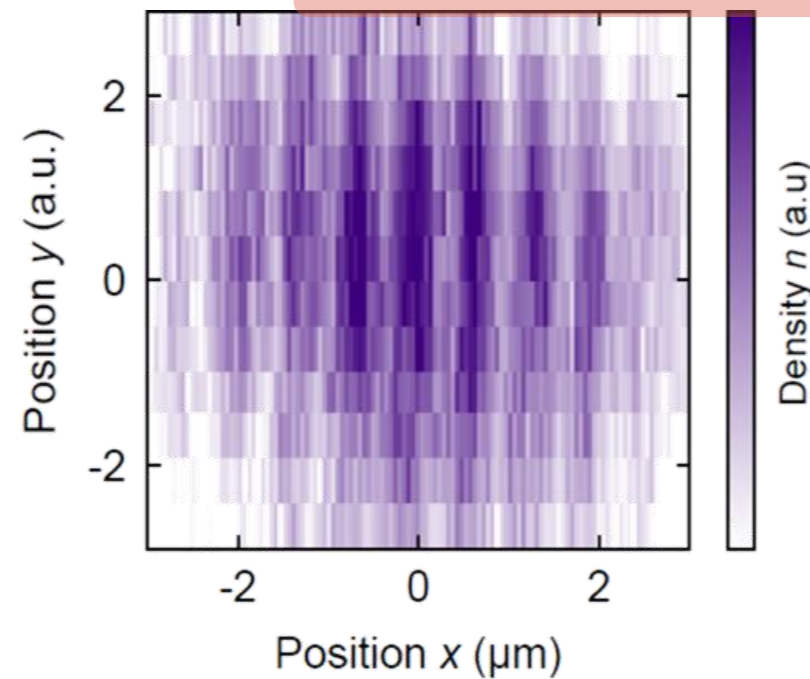
Raman coupling



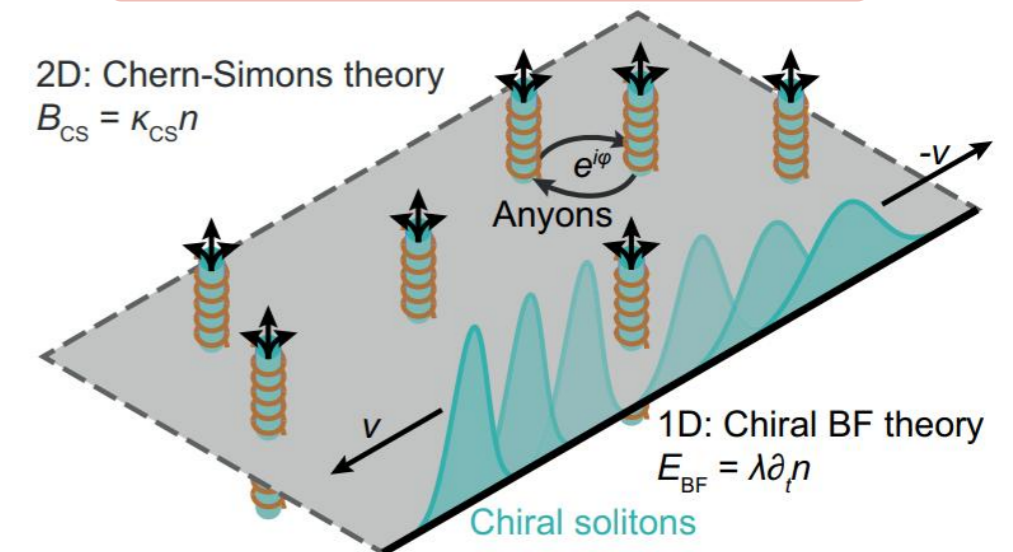
Supersolid dynamics



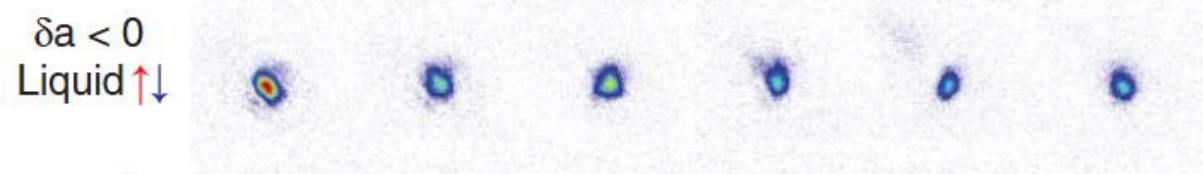
Stripe phase



Chiral BF theory



Droplets



Quantum Gases Experiment + Theory



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Epiquant

