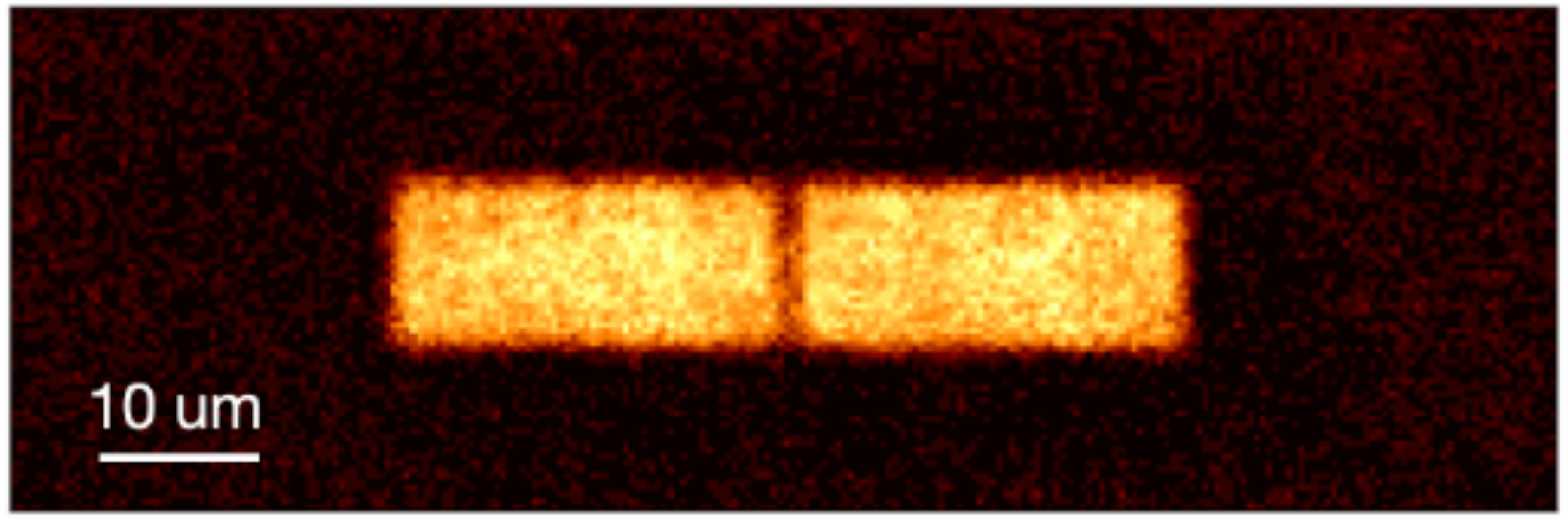
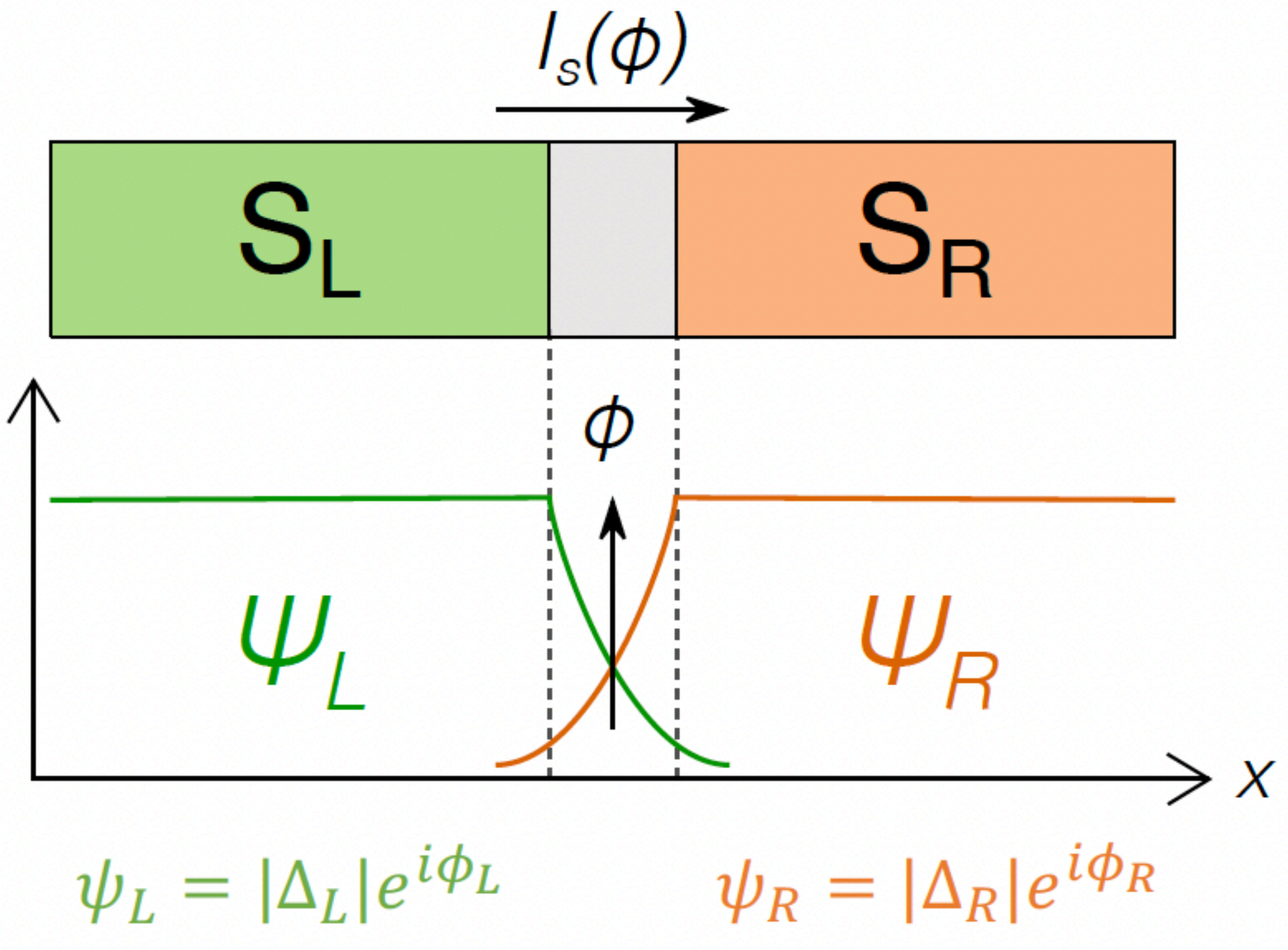
JOSEPHSON JUNCTIONS WITH ULTRACOLD ATOMIC FERMI GASES

Giulia Del Pace

INRiM and LENS



JOSEPHSON JUNCTIONS



- ☑ Thin barrier
- ☑ Small tunnelling probability

Josephson-Anderson equations:

$$I = I_c \sin(\phi)$$

$$\dot{\phi} = -\frac{2e}{\hbar} V$$

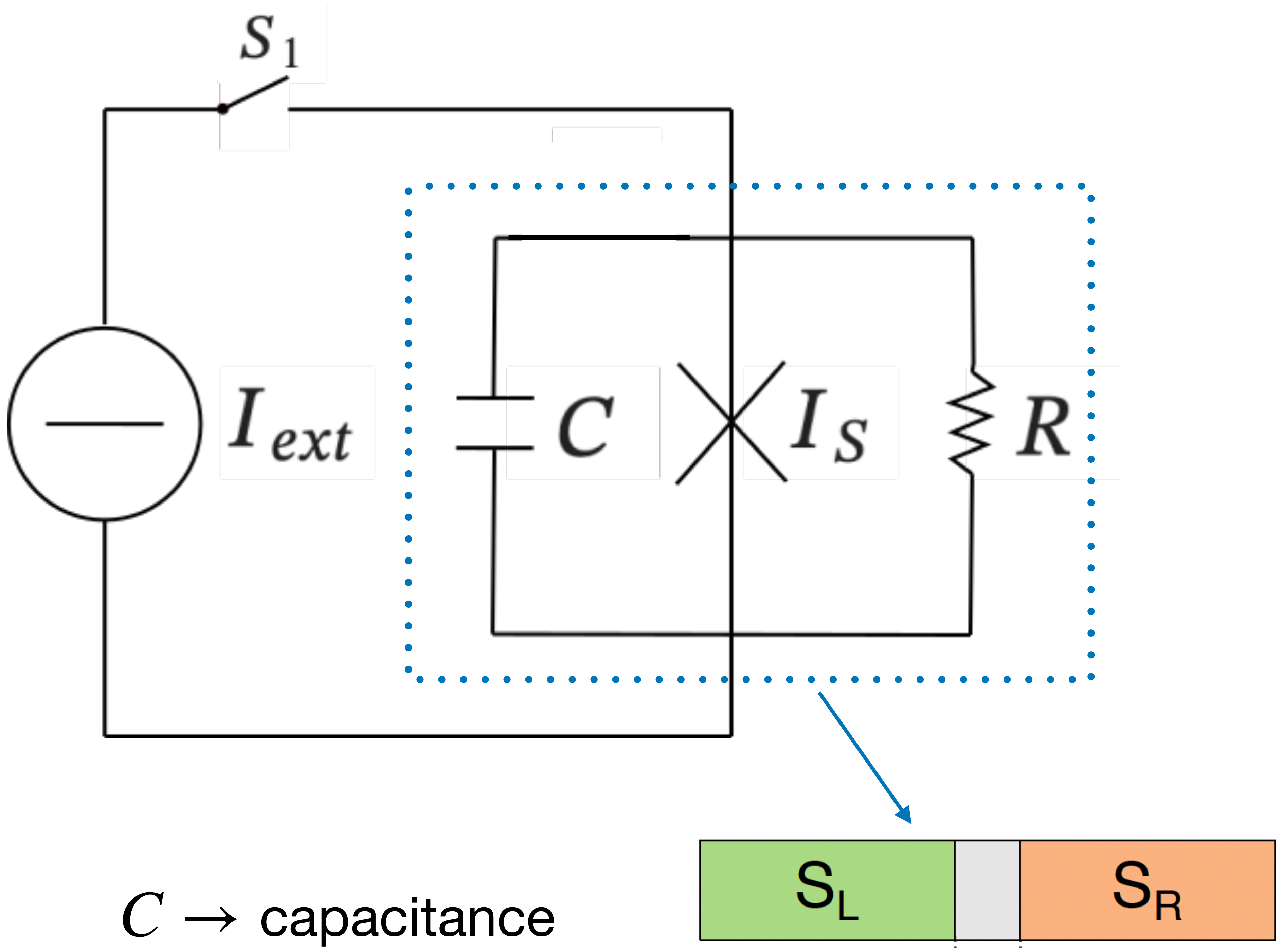
$\phi = \phi_L - \phi_R \rightarrow$ relative phase

$I_c \rightarrow$ critical current

$V \rightarrow$ potential difference

B. D. Josephson, Phys. Lett. 1, 251 (1962).
P. W. Anderson, Lectures on The Many-Body Problems, vol. 2 (Elsevier, 1964).

REAL JOSEPHSON JUNCTION: RCSJ MODEL



$C \rightarrow$ capacitance

$R \rightarrow$ resistance

$G = 1/R \rightarrow$ conductance

For the Kirchoff law we can write the following equations:

$$I_{ext} = I_c \sin(\phi) + GV + C\dot{V}$$

$$\dot{\phi} = \frac{2e}{\hbar} V$$

They can be combined in a single differential equation for ϕ:

$$\frac{C\hbar}{2e}\ddot{\phi} = I_{ext} - I_C \sin(\phi) - \frac{G\hbar}{2e}\dot{\phi}$$

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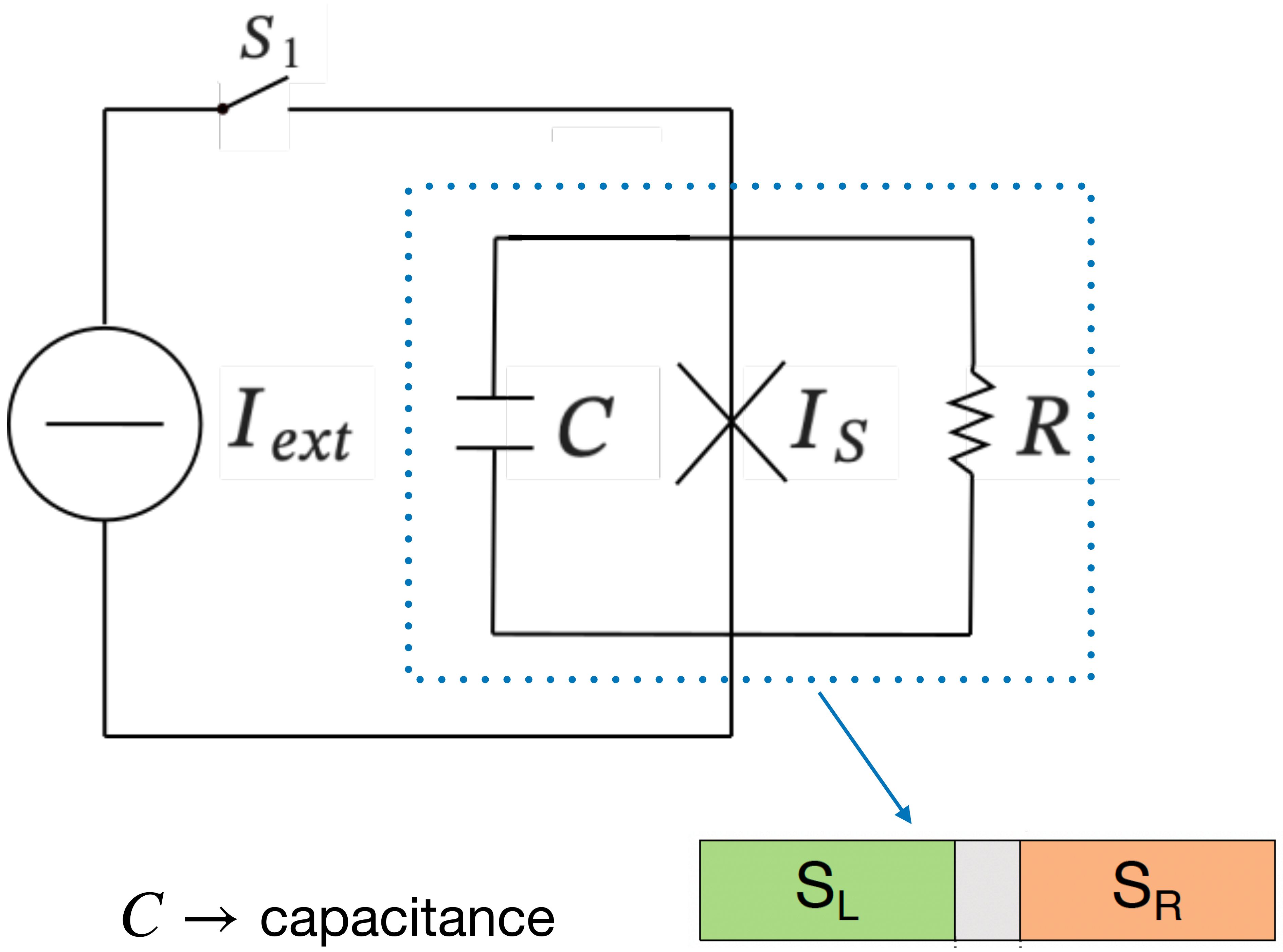

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Mass

Conservative force

Viscous force

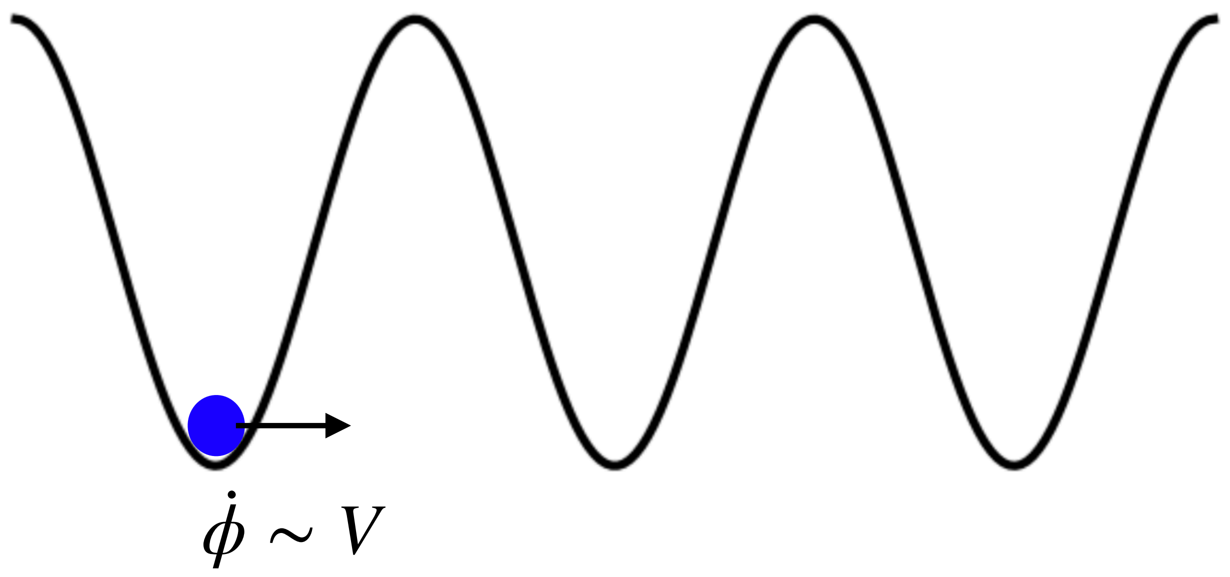
Equation of motion for a phase particle of mass ~C subject to a viscous force ~G in the **washboard potential**:

$$U(\phi) = -I_{ext}\phi + I_c \cos(\phi)$$

AC AND DC JOSEPHSON EFFECT IN THE WASHBOARD POTENTIAL

$$U(\phi) = -I_{ext}\phi + I_c \cos(\phi)$$

AC Josephson effect



$$I_{ext} = 0$$

$$V \neq 0$$

The phase particle oscillates around one of the minima of the washboard potential: **Josephson oscillation** of V and ϕ in counter-phase

AC AND DC JOSEPHSON EFFECT IN THE WASHBOARD POTENTIAL

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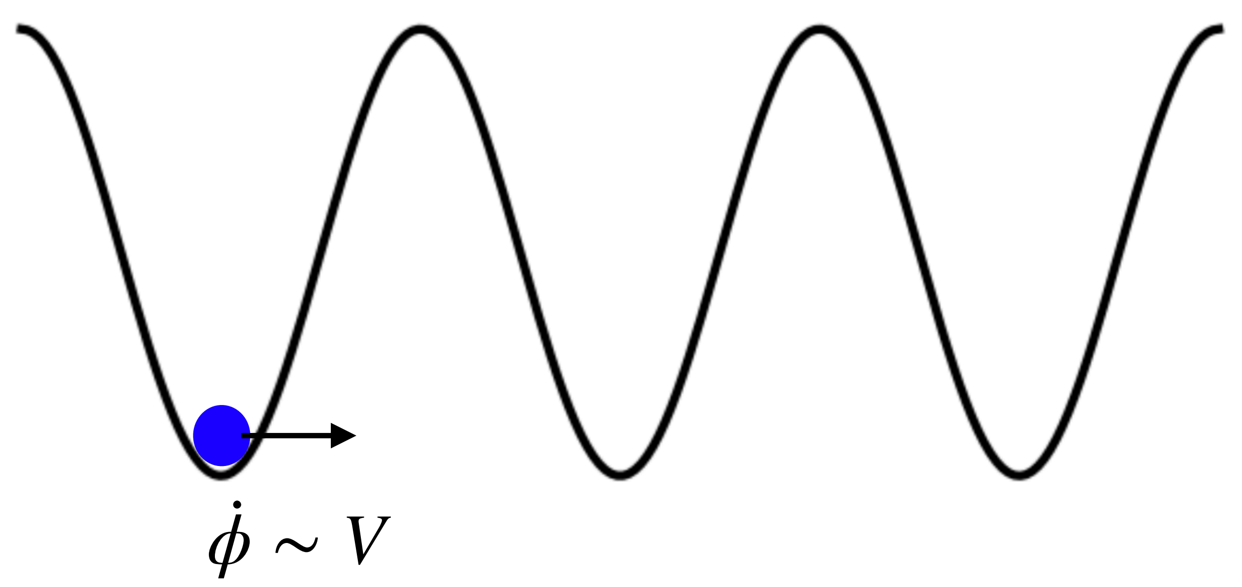

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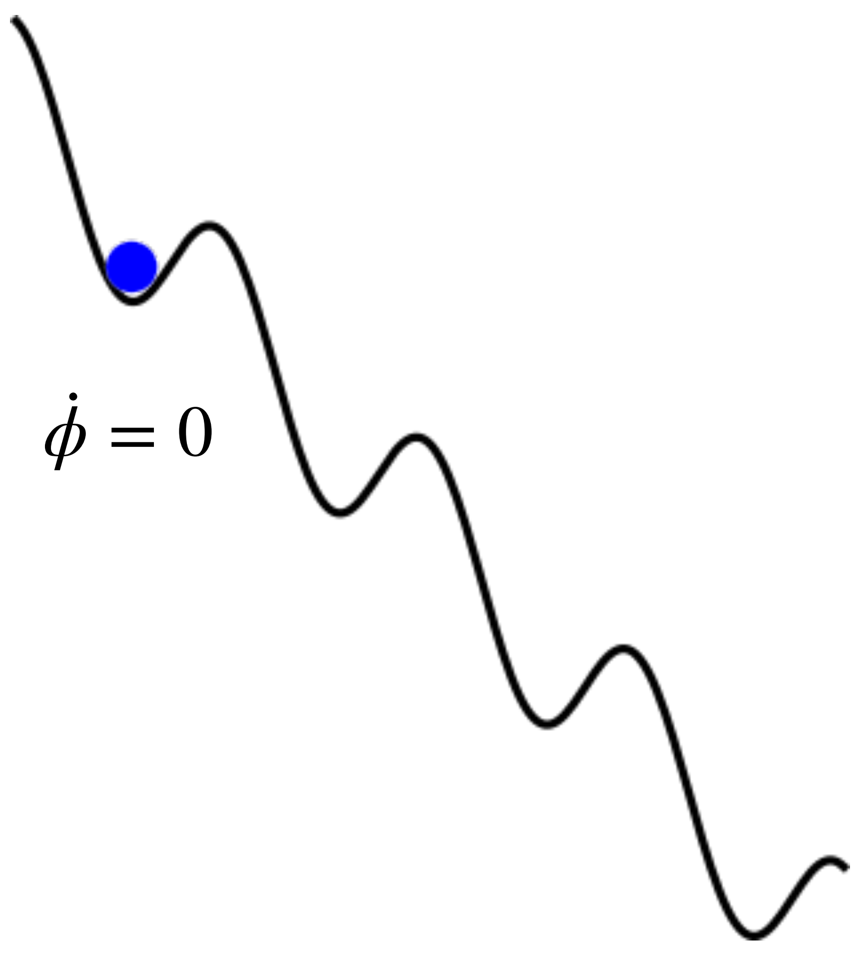
DC Josephson effect

$I_{ext} < I_c$



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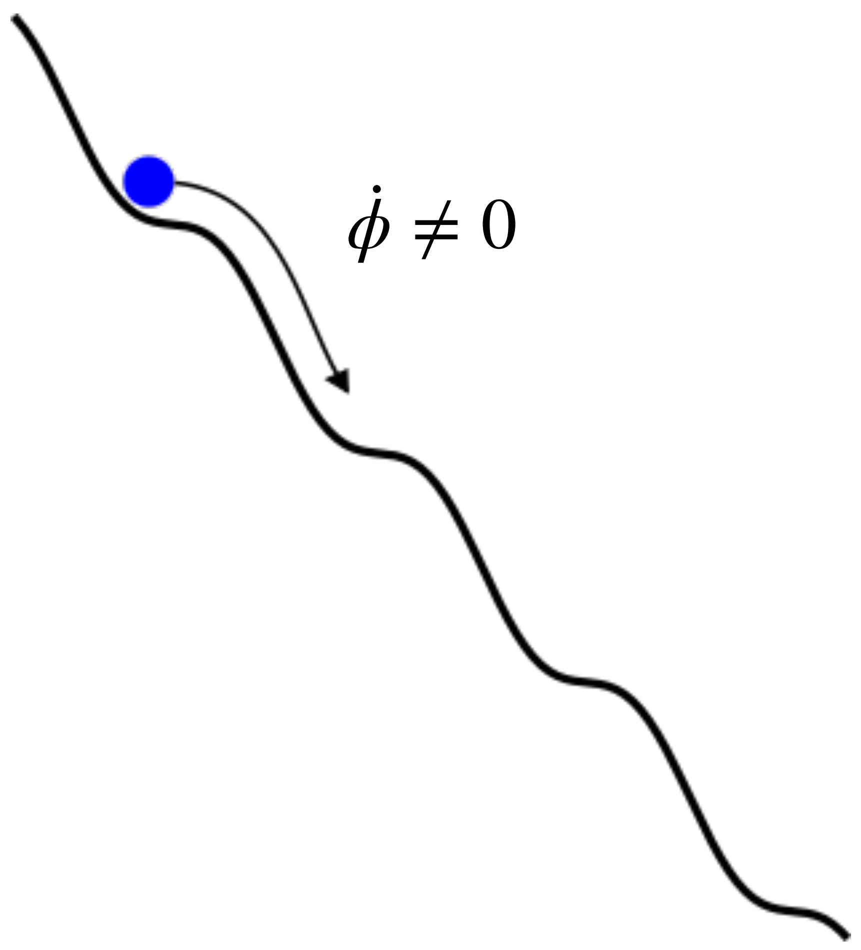
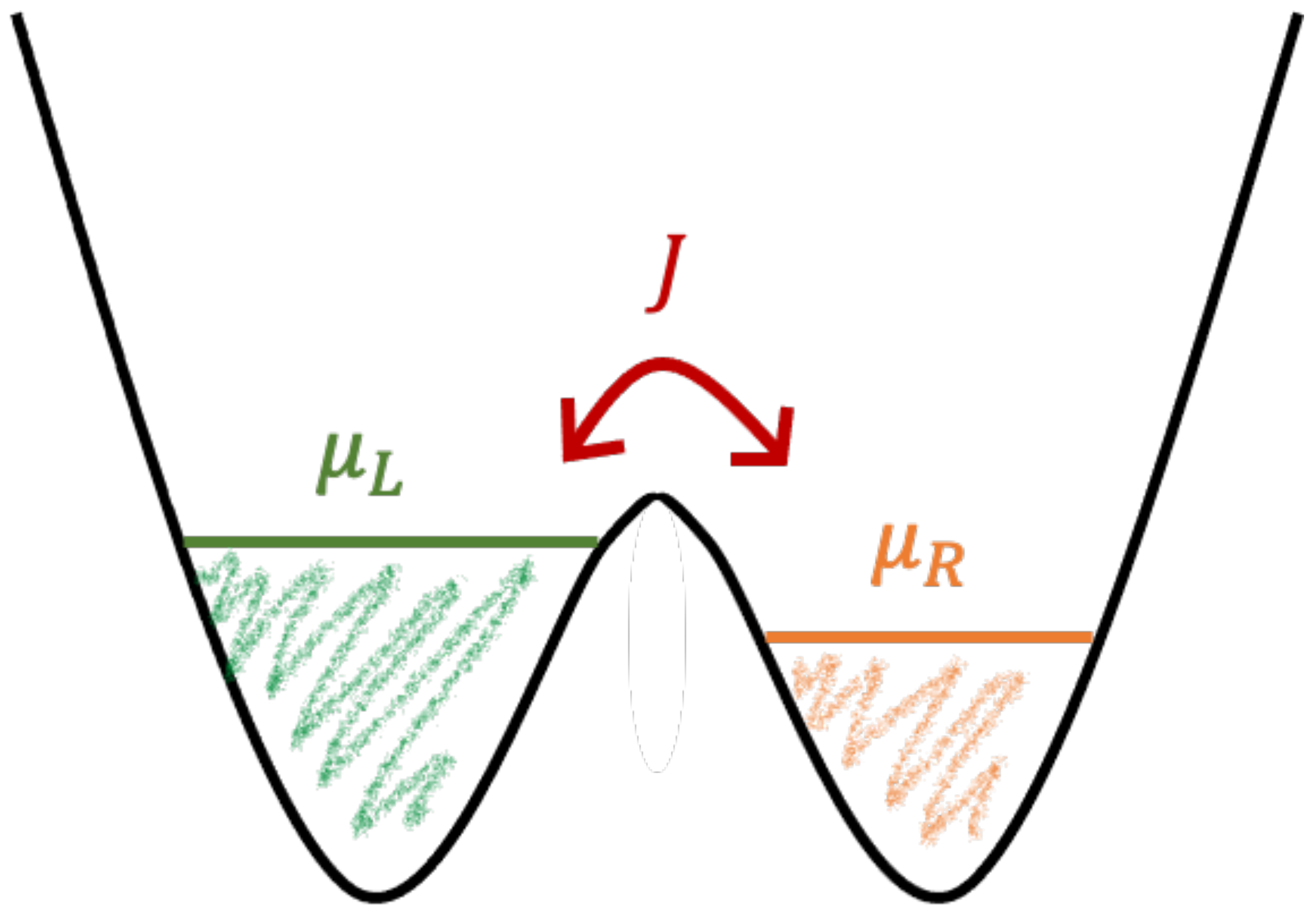


For $I_{ext} < I_c$ the phase particle is still in one of the minima of the washboard potential: a **dissipationless current** flows trough the junction.

JOSEPHSON JUNCTION WITH ULTRA COLD ATOMS



$\psi_L = |\psi_L| e^{i\phi_L}$ $\psi_R = |\psi_R| e^{i\phi_R}$

- ☑ Thin barrier
- ☑ Small tunnelling probability

Josephson-Anderson equations:

$$I = I_c \sin(\phi)$$

$$\dot{\phi} = -\frac{\Delta\mu}{\hbar}$$

$\Delta\mu = \mu_L - \mu_R \rightarrow$ chemical potential difference

$\phi = \phi_L - \phi_R \rightarrow$ relative phase

$z = (N_L - N_R)/(N_L + N_R) \rightarrow$ particle imbalance

$I = \dot{z} \rightarrow$ particle current

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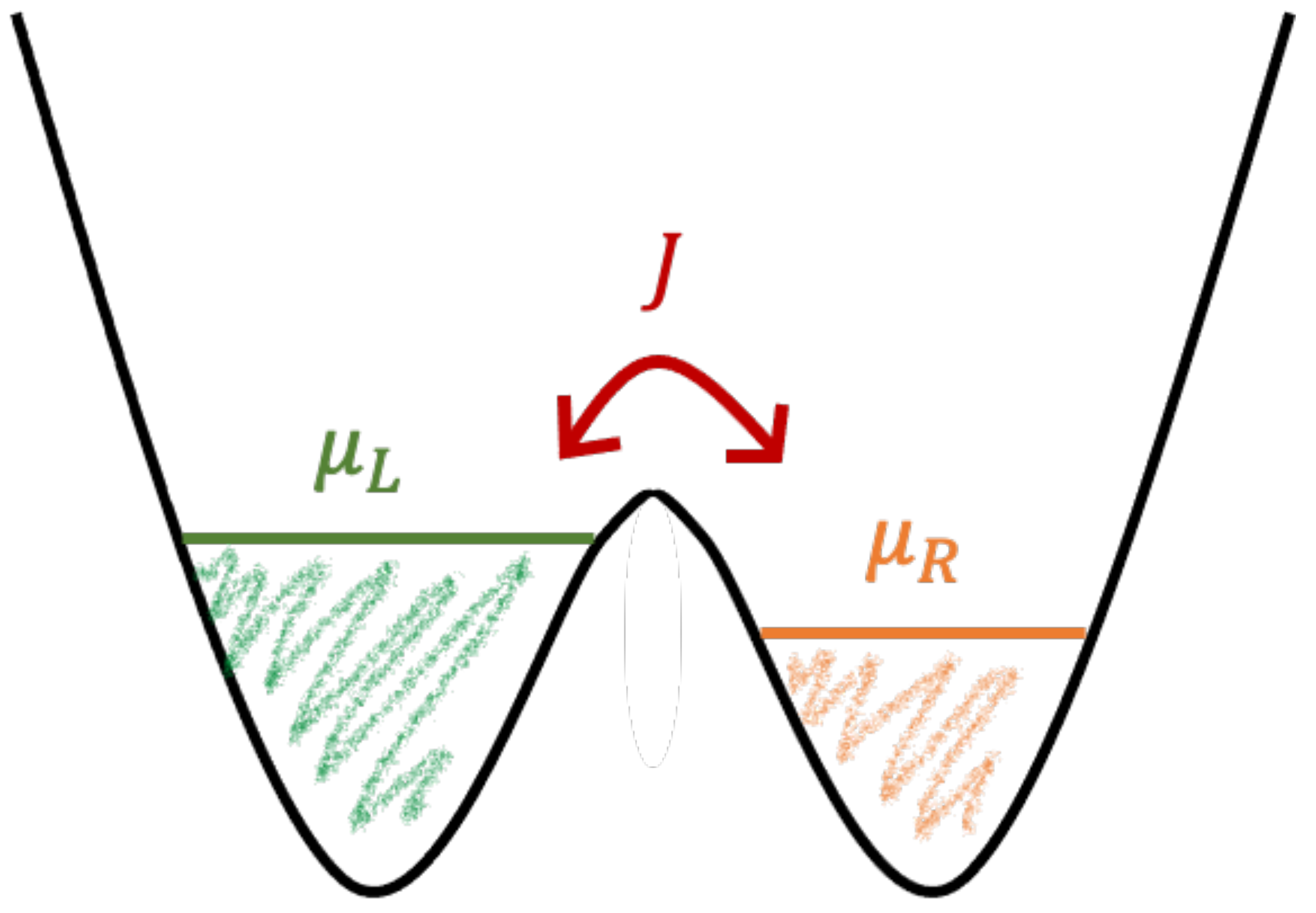

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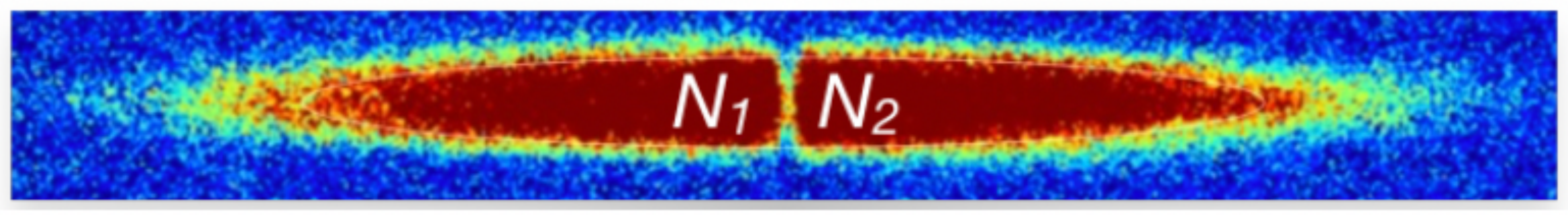
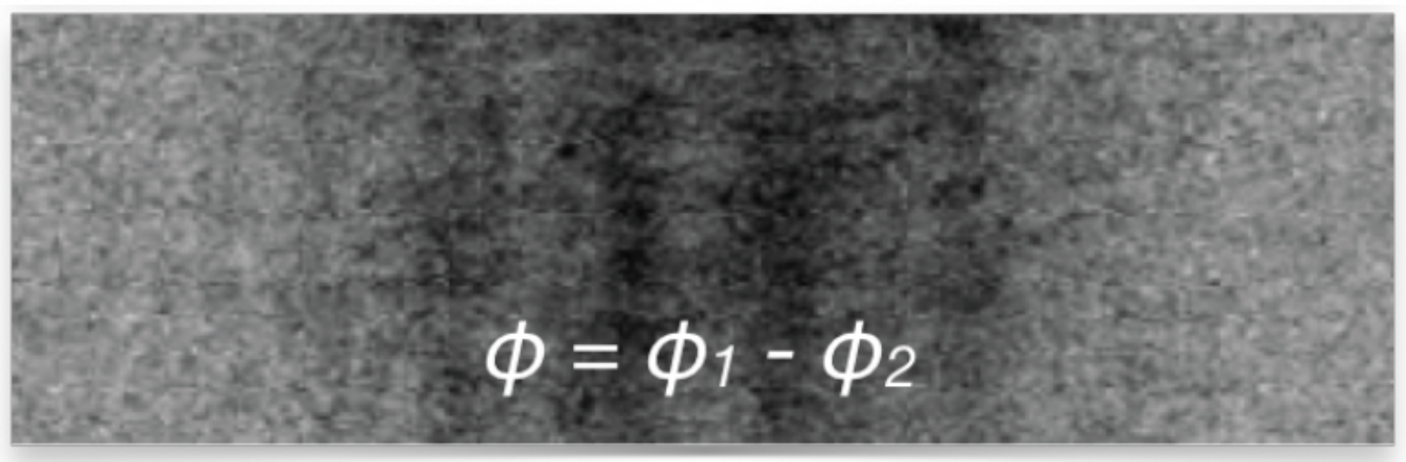
Observables



In situ imaging

number of particle imbalance

$z \rightarrow \Delta\mu$



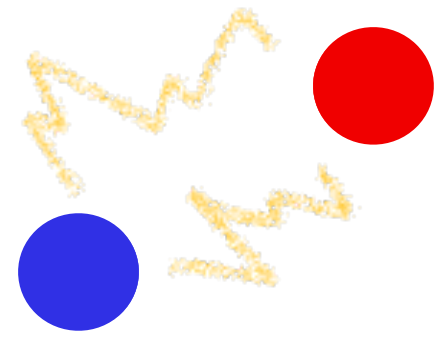
Time-of-flight imaging

relative phase ϕ

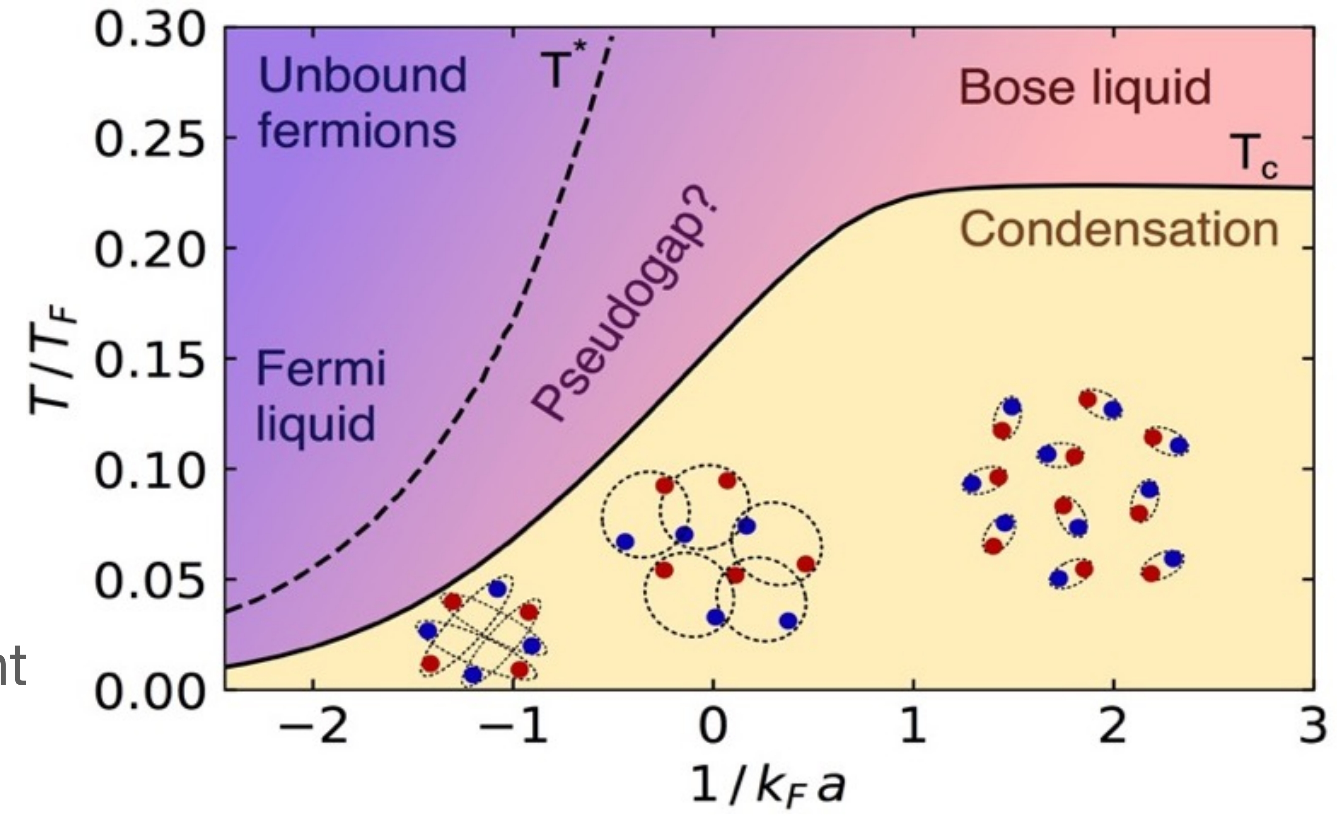
G. Valtolina et al., Science 350.6267 (2015)

FERMIONIC SUPERFLUIDS IN ARBITRARY GEOMETRIES

BEC-BCS crossover



$a \rightarrow$ scattering lenght

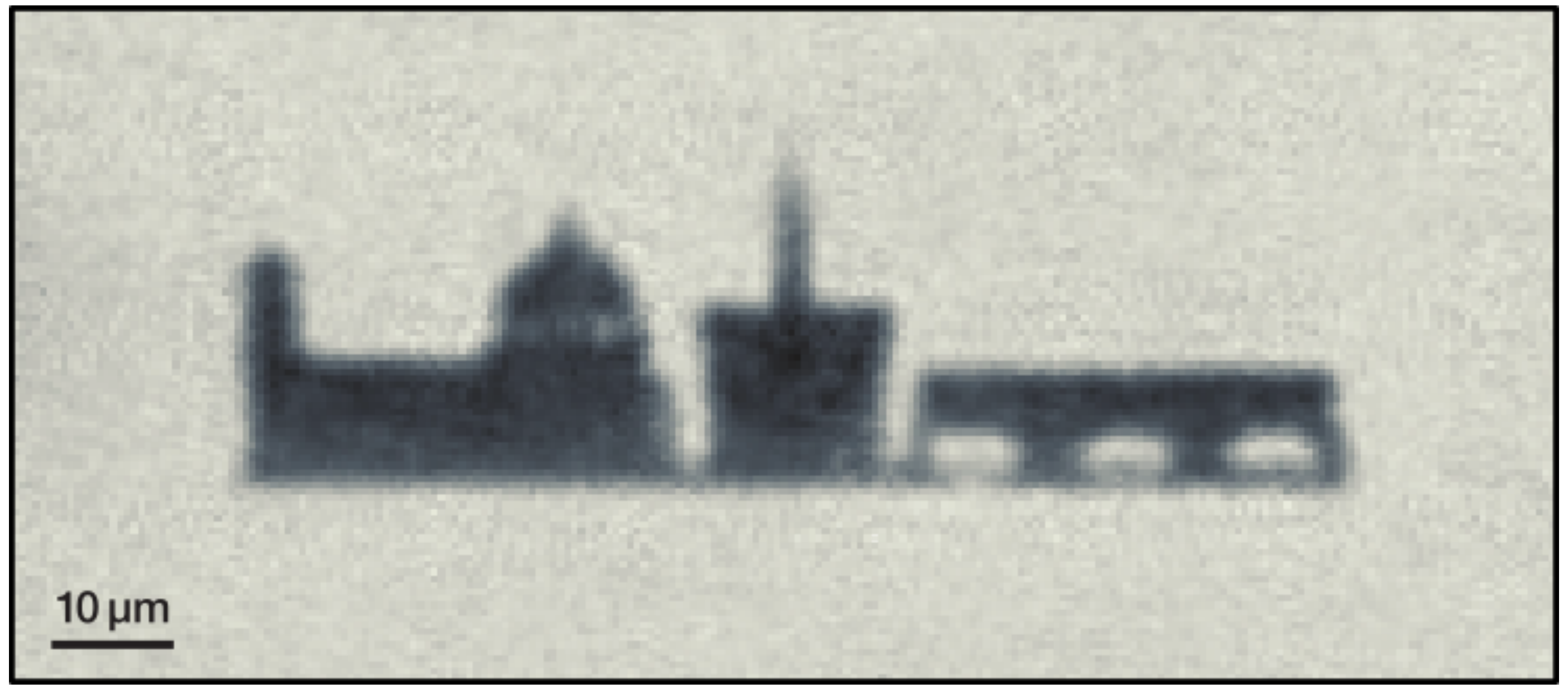


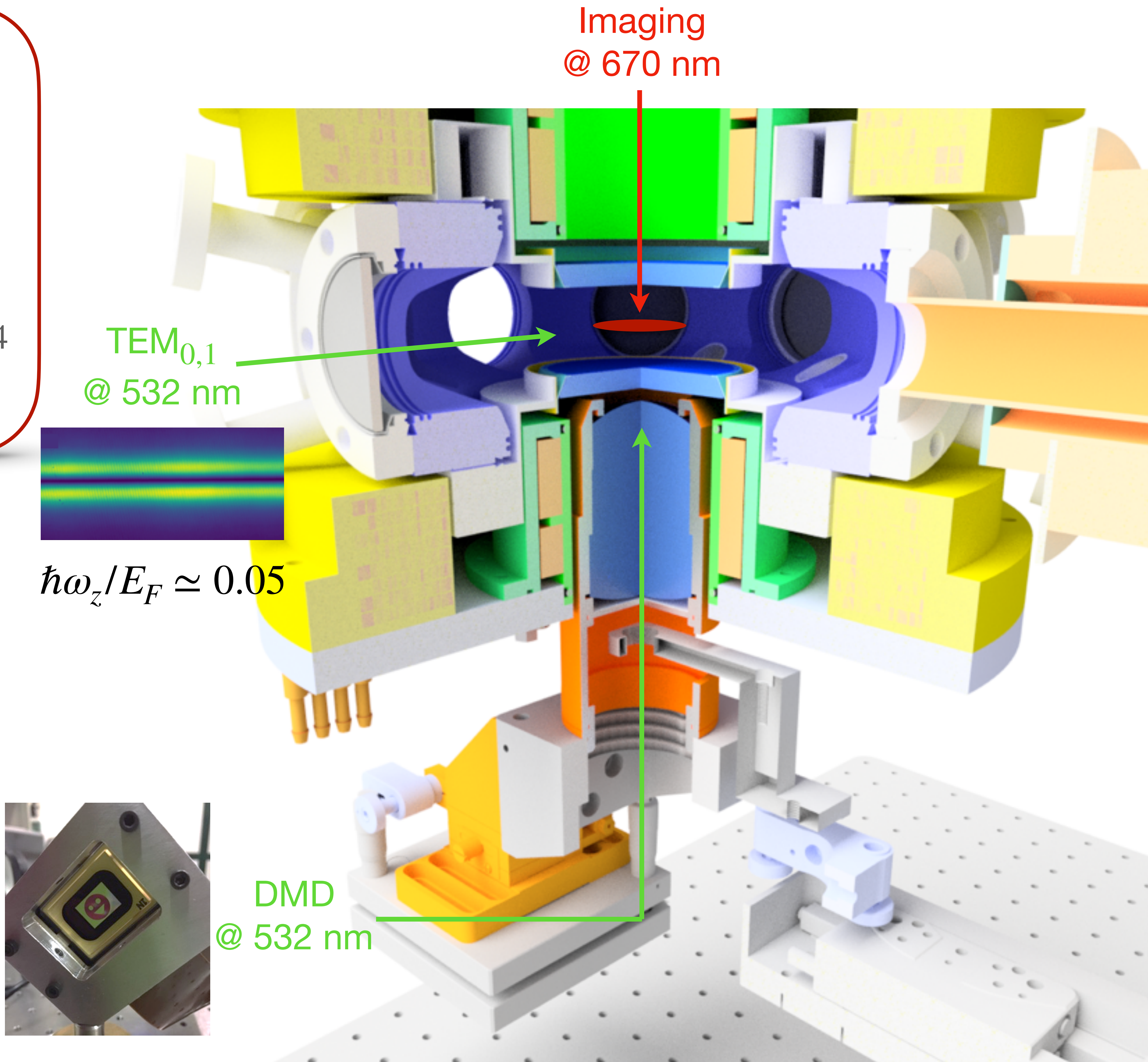
BEC
$1/k_F a = 3.6$

UFG
$1/k_F a = 0$

BCS
$1/k_F a = -0.4$

Arbitrary optical potential on a micron scale





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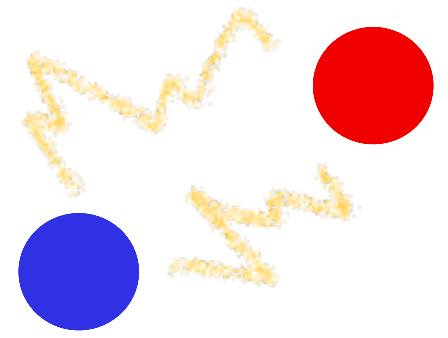

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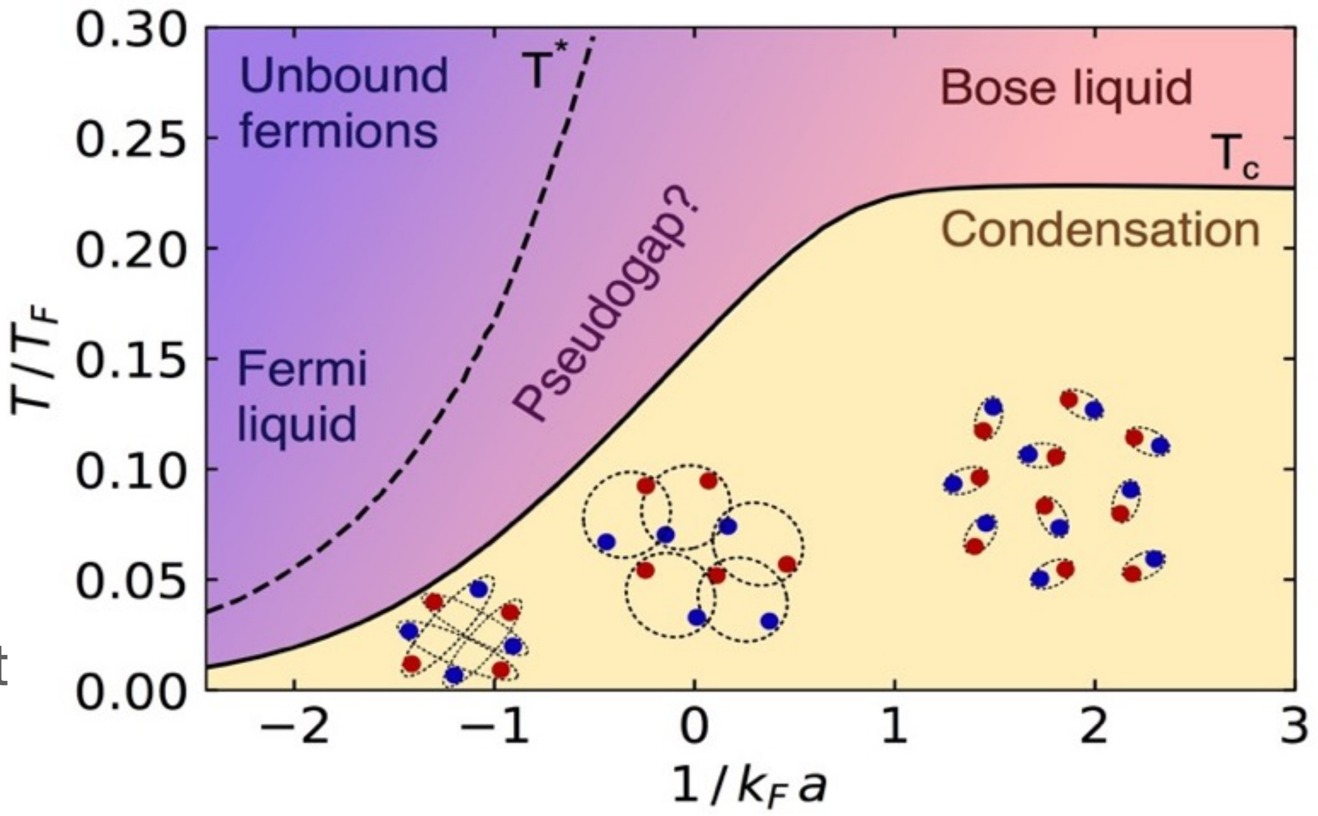

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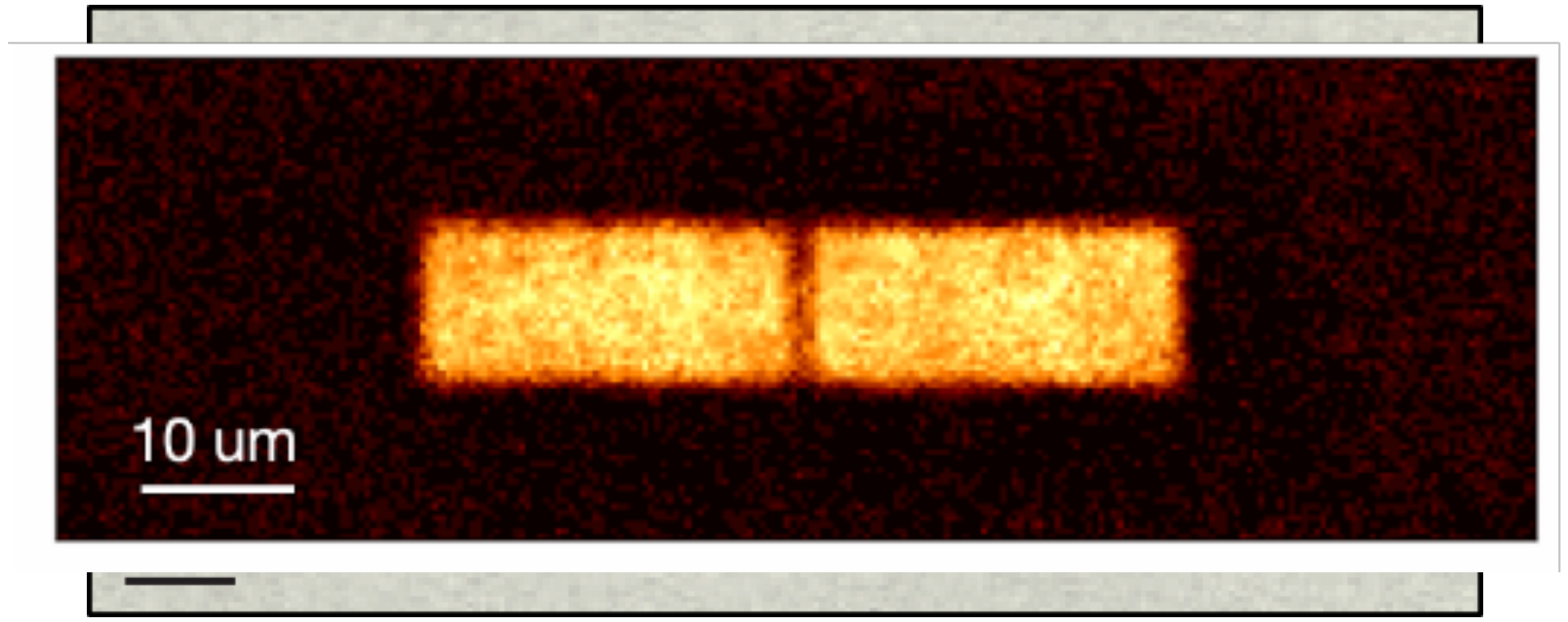
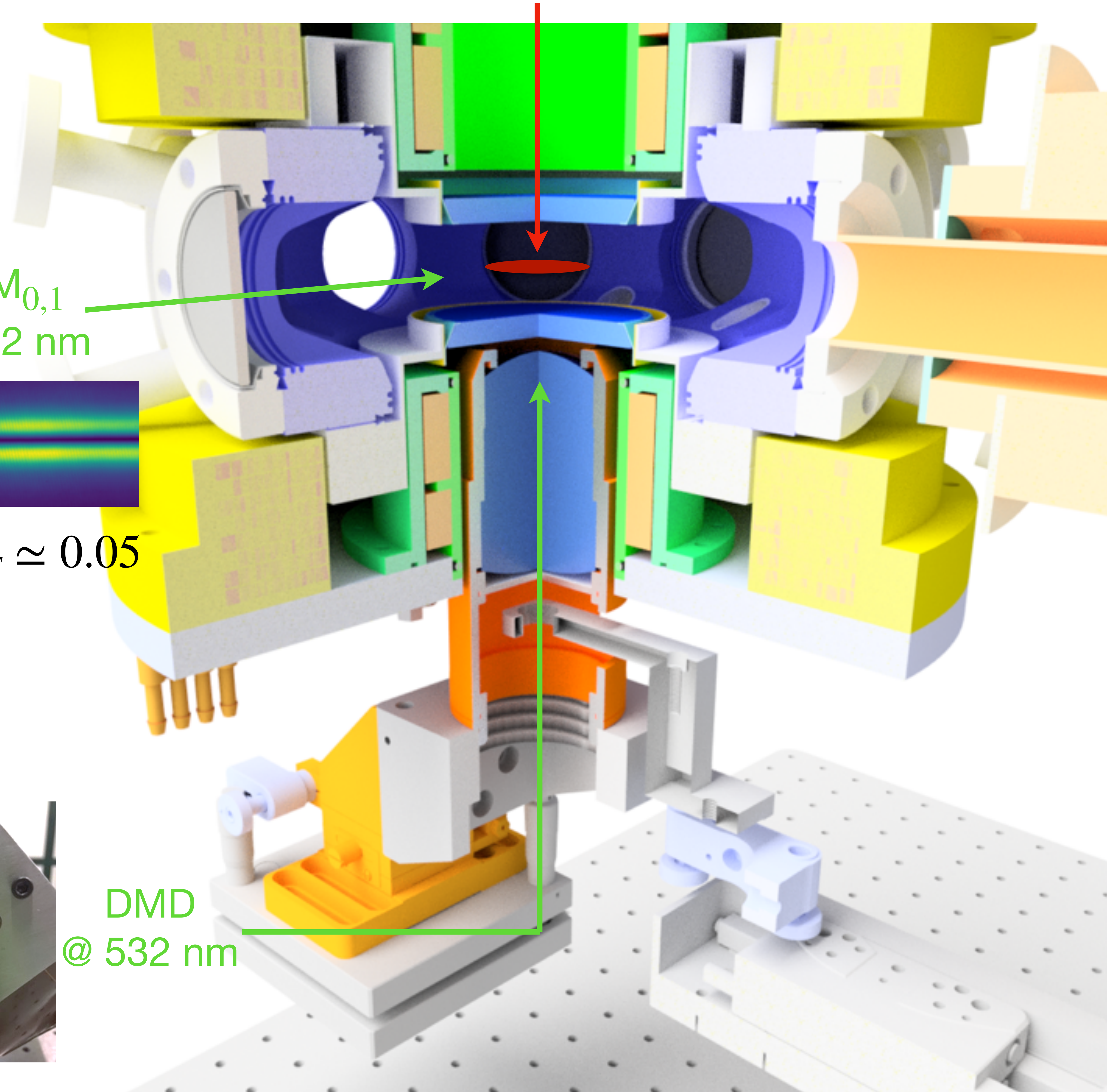
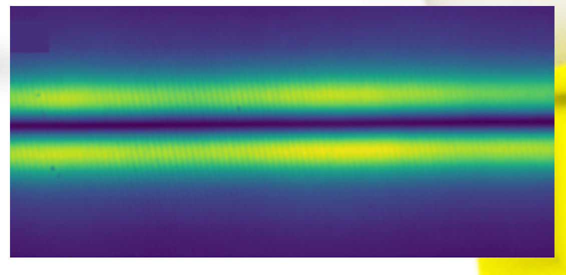
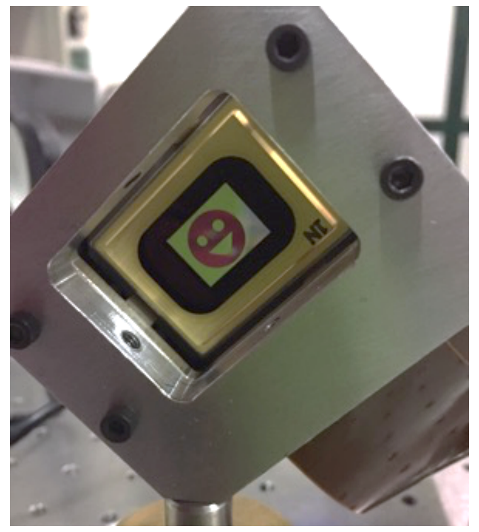


Imaging
@ 670 nm

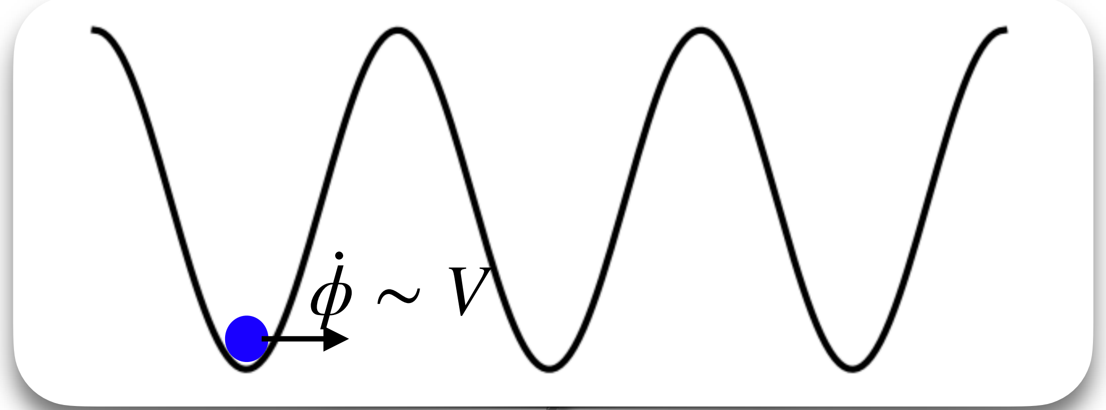




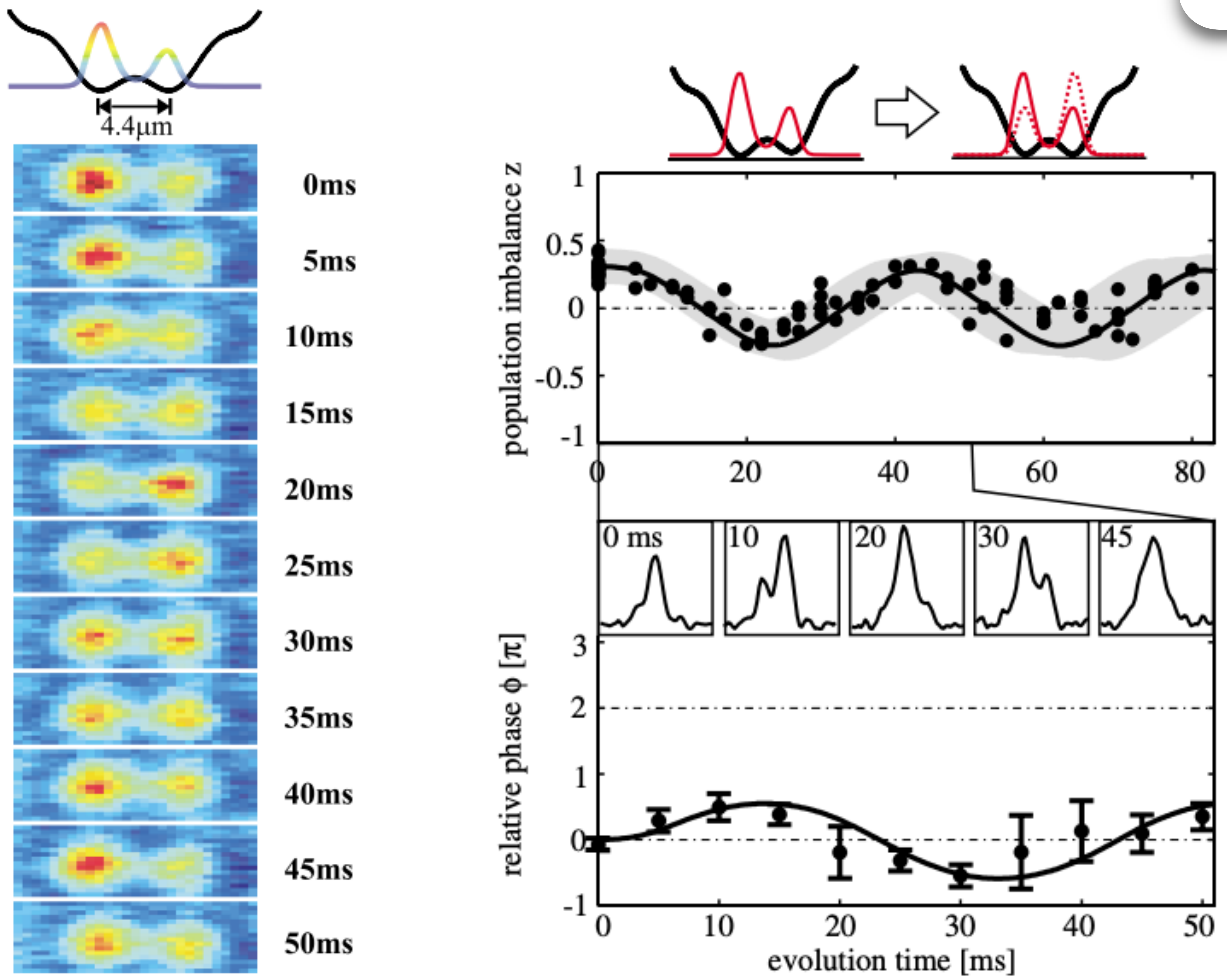
$\hbar\omega_z / E_F \simeq 0.05$



AC JOSEPHSON EFFECT WITH ATOMIC JUNCTIONS



Bosonic superfluids



Michael Albiez, et al., Phys. Rev. Lett. **95**, 010402 (2005)

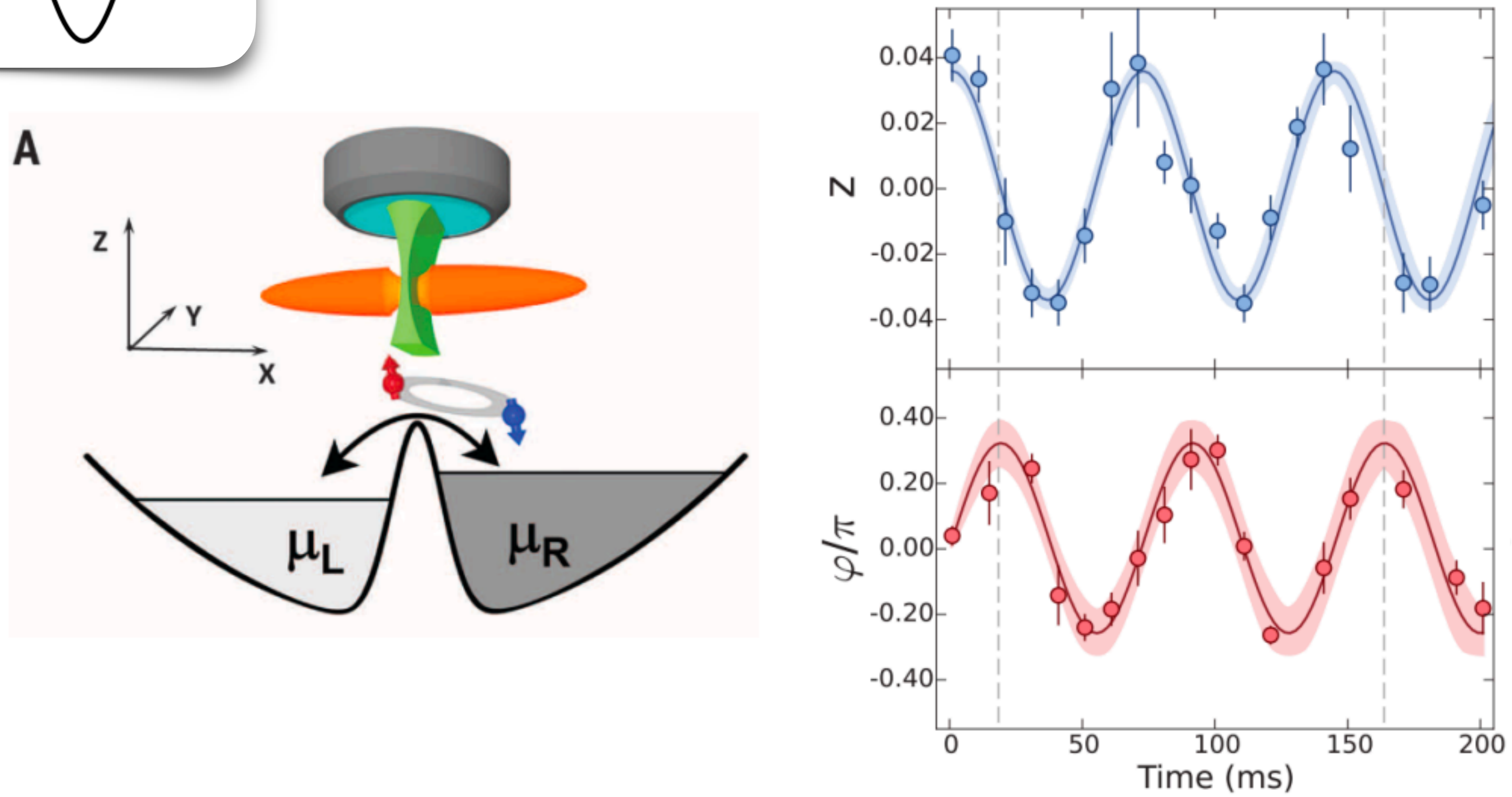
<u>See also:</u>

F. S. Cataliotti, et al. . Science 293.5531: 843-846 (2001)

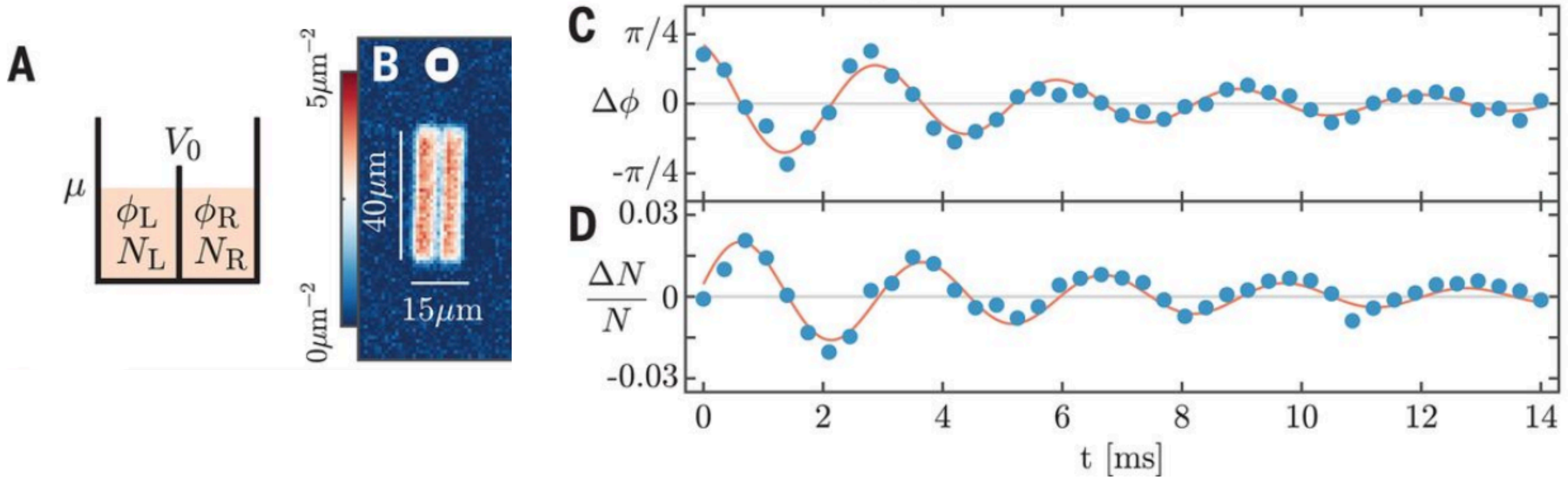
G. Spagnolli, et al. Phys. Rev. Lett. 118.23 230403 (2017)

G. Biagioni, et al., Nature 629.8013 773-777 (2024)

Fermionic superfluids



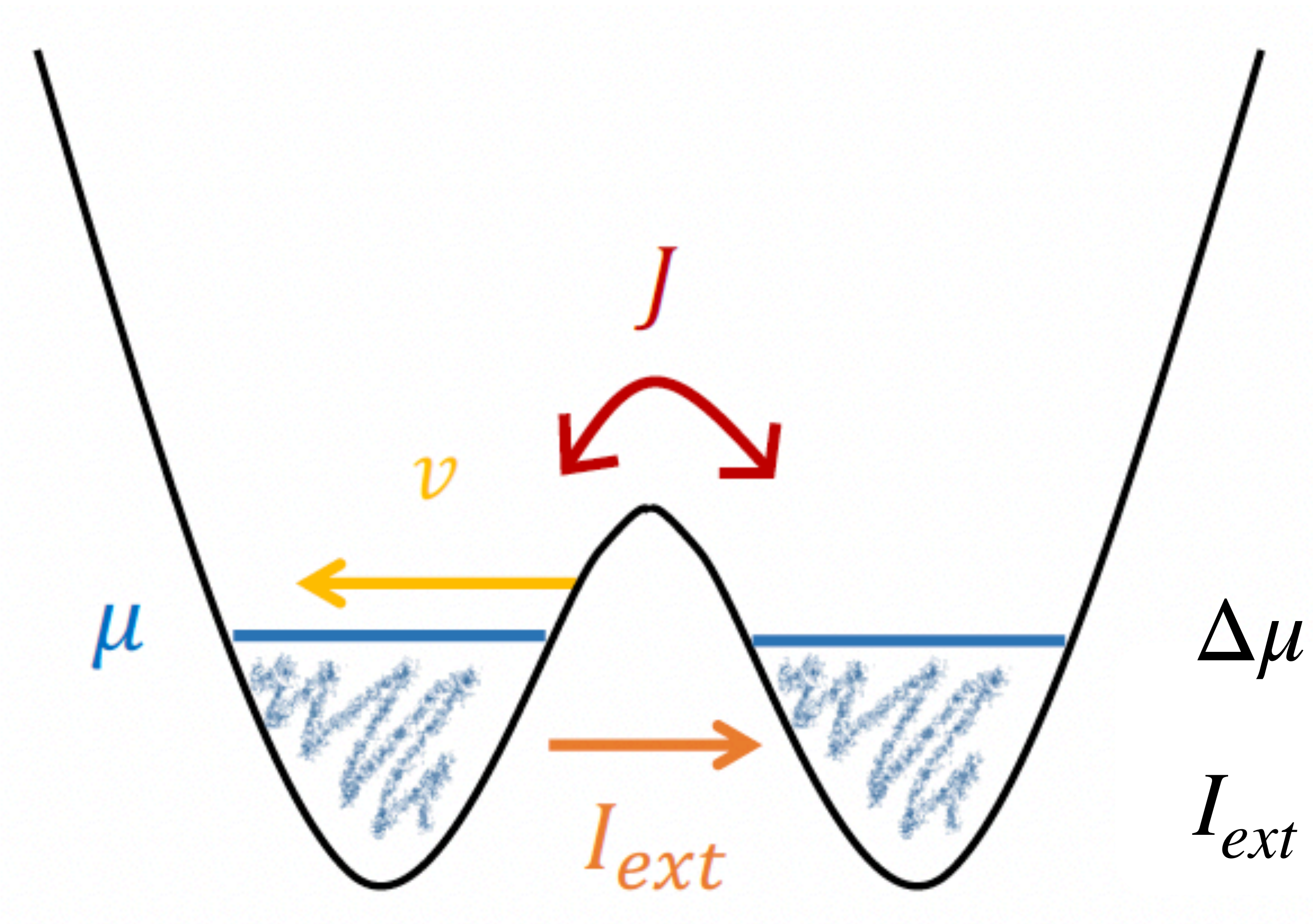
G. Valtolina et al., *Science* **350**.6267 (2015)



N. Luick *et al., Science* **369**,89-91(2020).

DC JOSEPHSON EFFECT WITH ATOMIC JUNCTION

How to inject a current in atomic Josephson junctions?



$\Delta\mu = 0$

$I_{ext} \sim -v$

We can inject a current in the atomic JJ by **moving the tunnelling barrier** with a given velocity

Theoretical proposal:
Giovanazzi et al., Phys. Rev. Lett. 84 (2000)
Realizations with bosonic superfluids:
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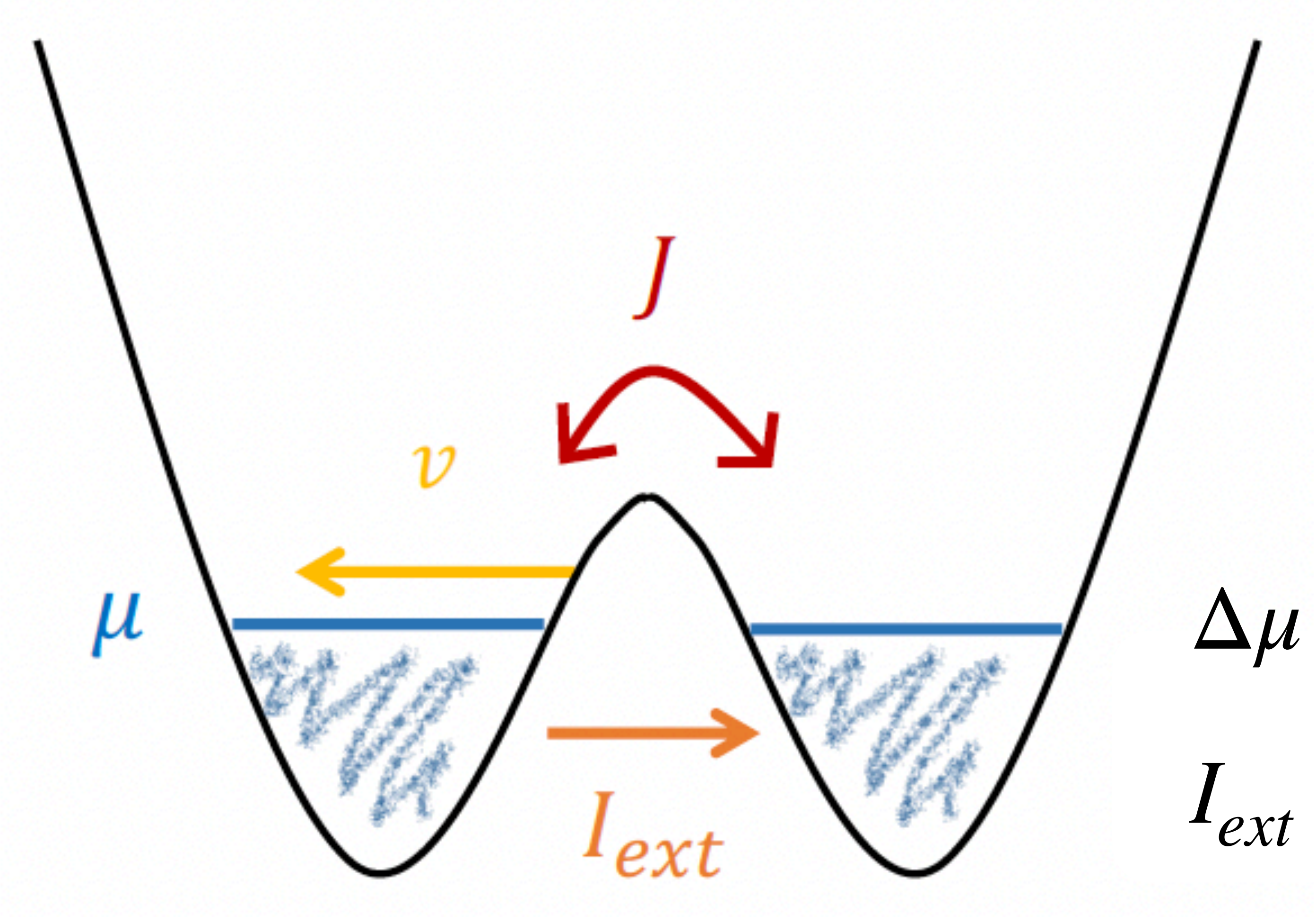

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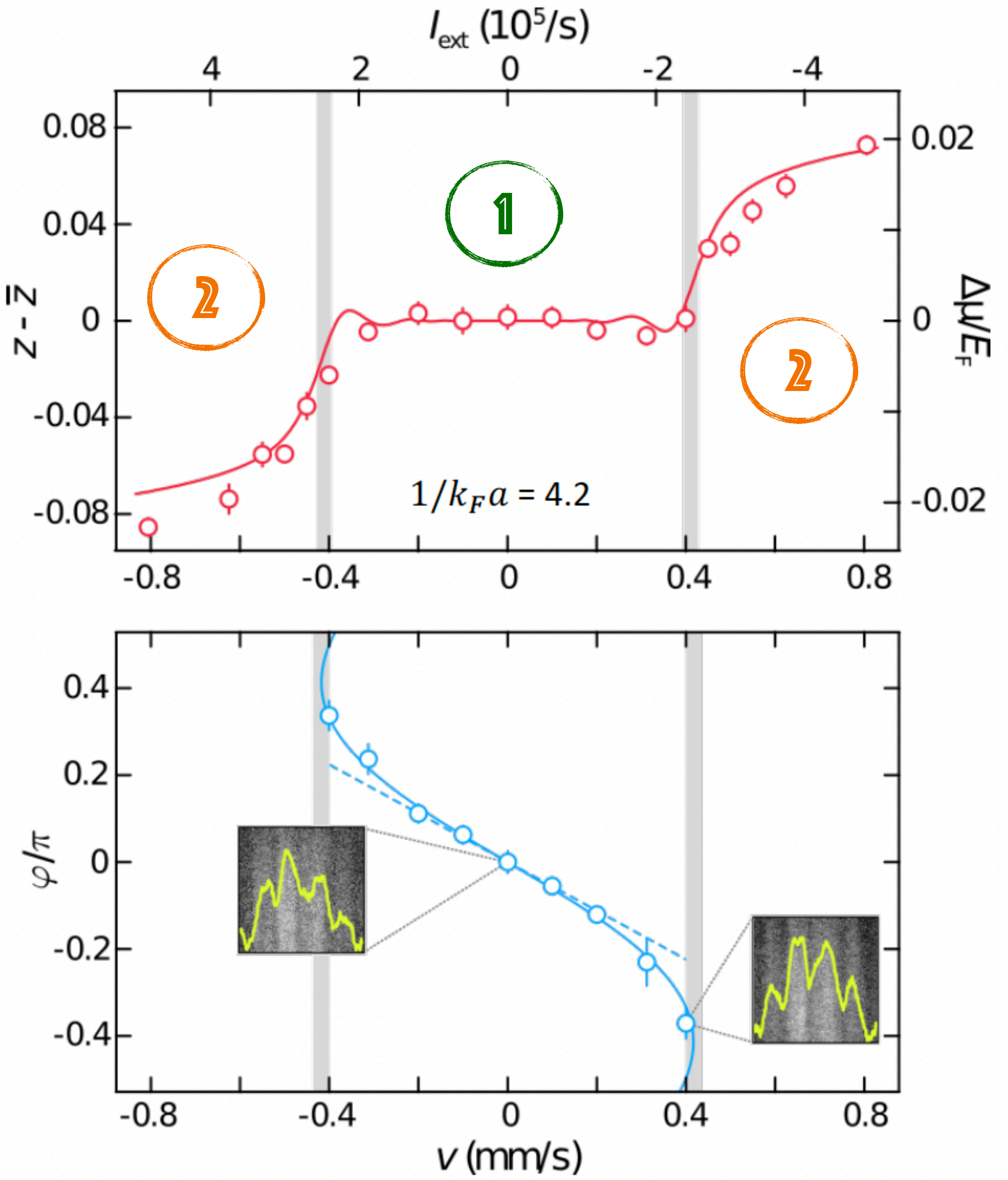
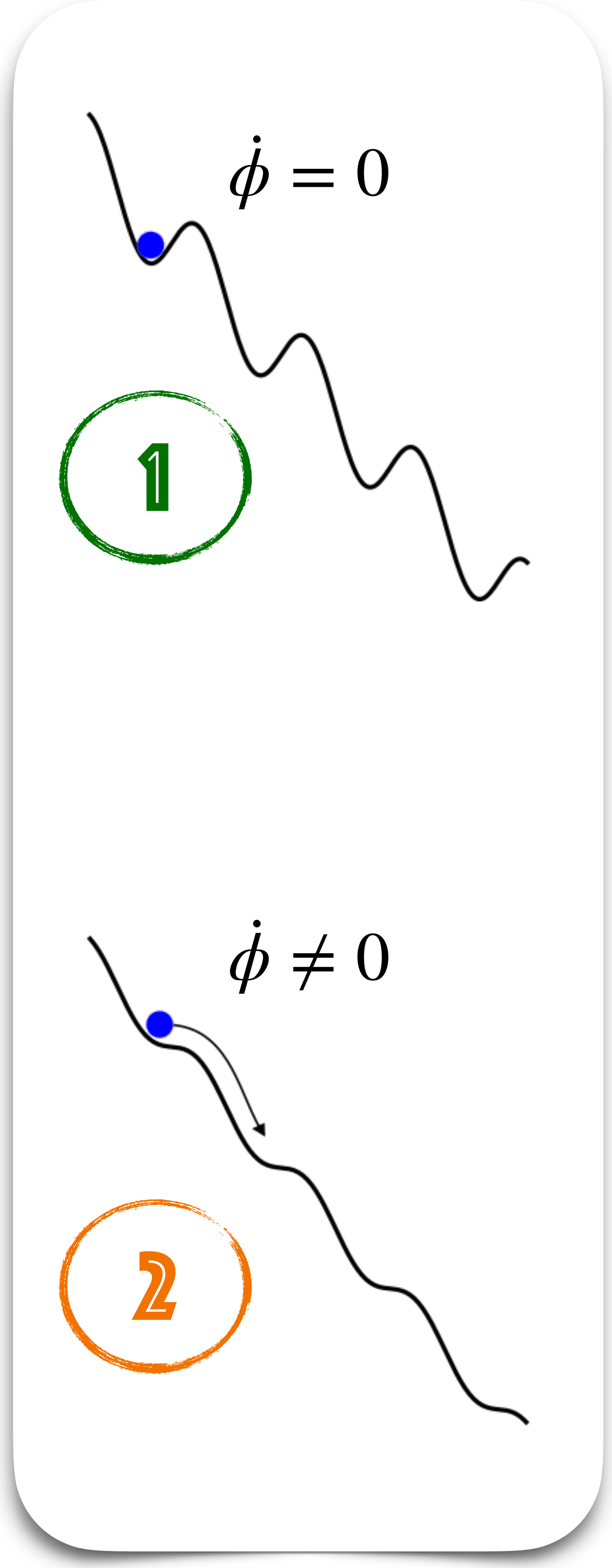
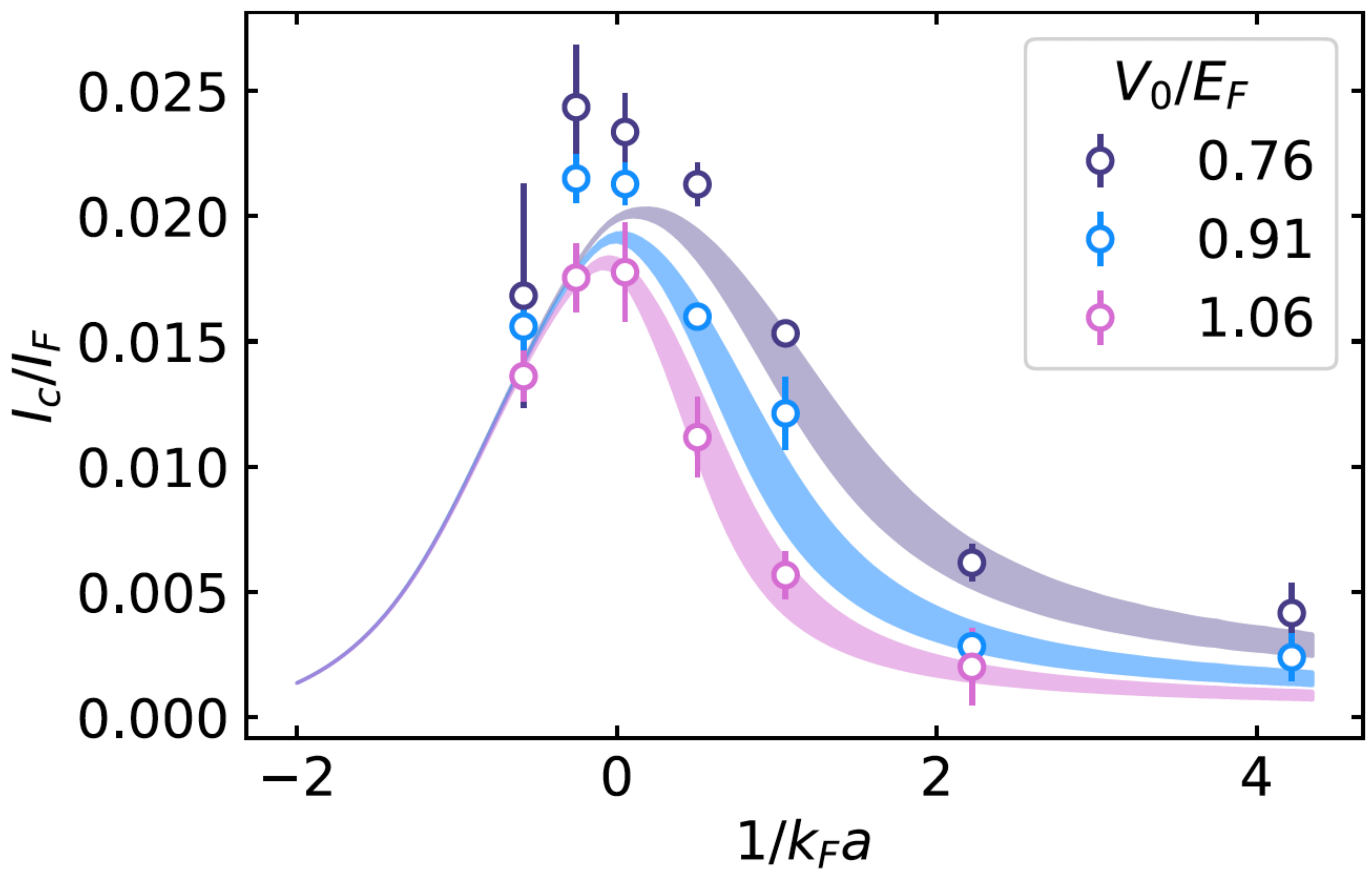


W. J. Kwon, et al., *Science*, 369, 6499 (2020)



CRITICAL CURRENT IN THE BEC-BCS CROSSOVER

W. J. Kwon, et al., *Science*, 369, 6499 (2020)



$$j_C = n_0 \frac{\mu}{2\sqrt{2m\mu}} \, | \, t(\mu) \, |$$

- Condensed fraction n_0
- Bulk properties $\sim \sqrt{\mu}$
- Tunneling probability amplitude $|\, t \,|$

F. Meier, W. Zwerger. " *Phys. Rev. A* 64.3 033610 (2001)
M. Zaccanti,, W. Zwerger. *Phys. Rev. A* 100.6 063601 (2019)

Similarly to the Ambegaokar-Baratoff relation $I_c \sim \Delta$

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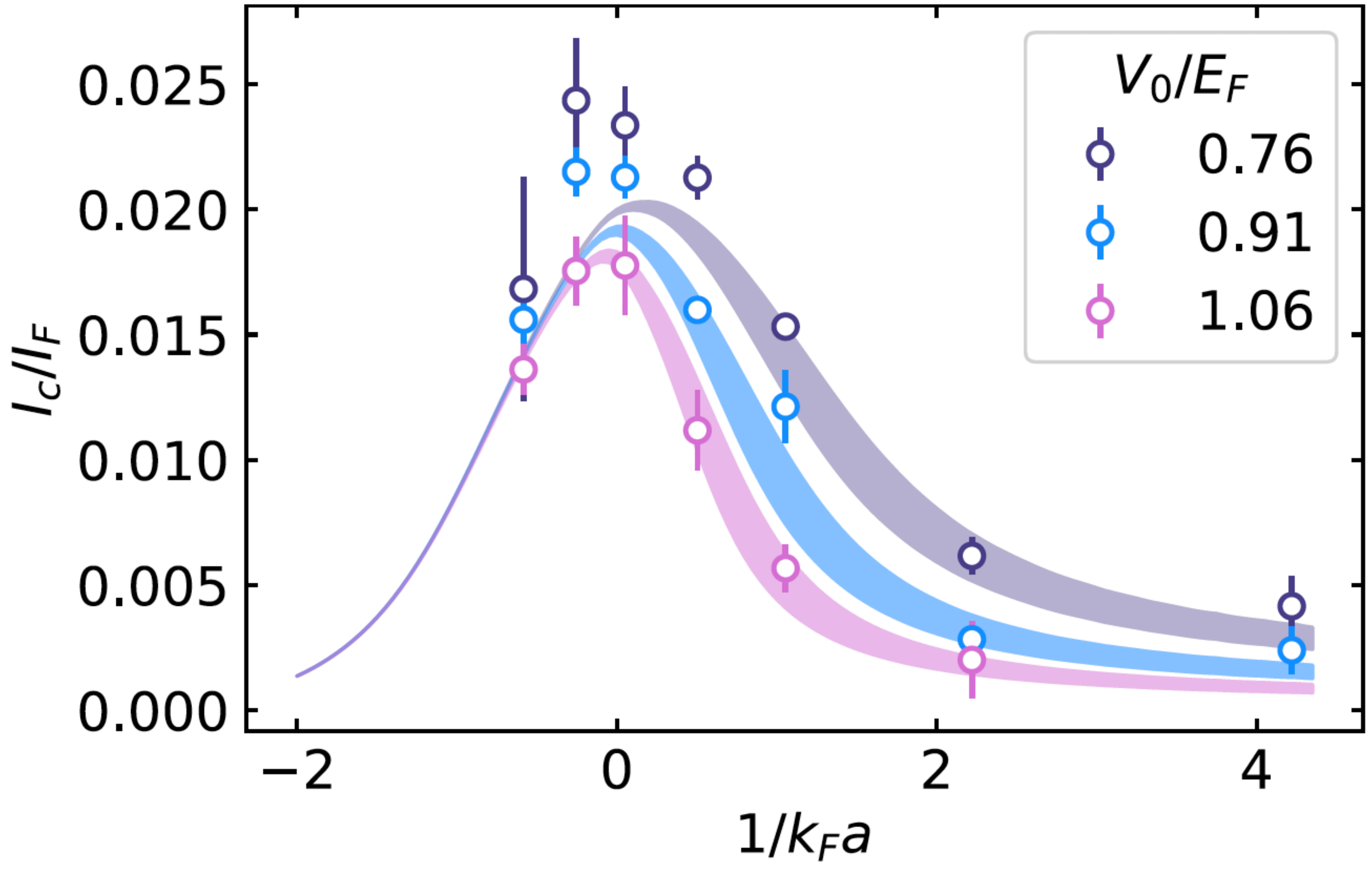

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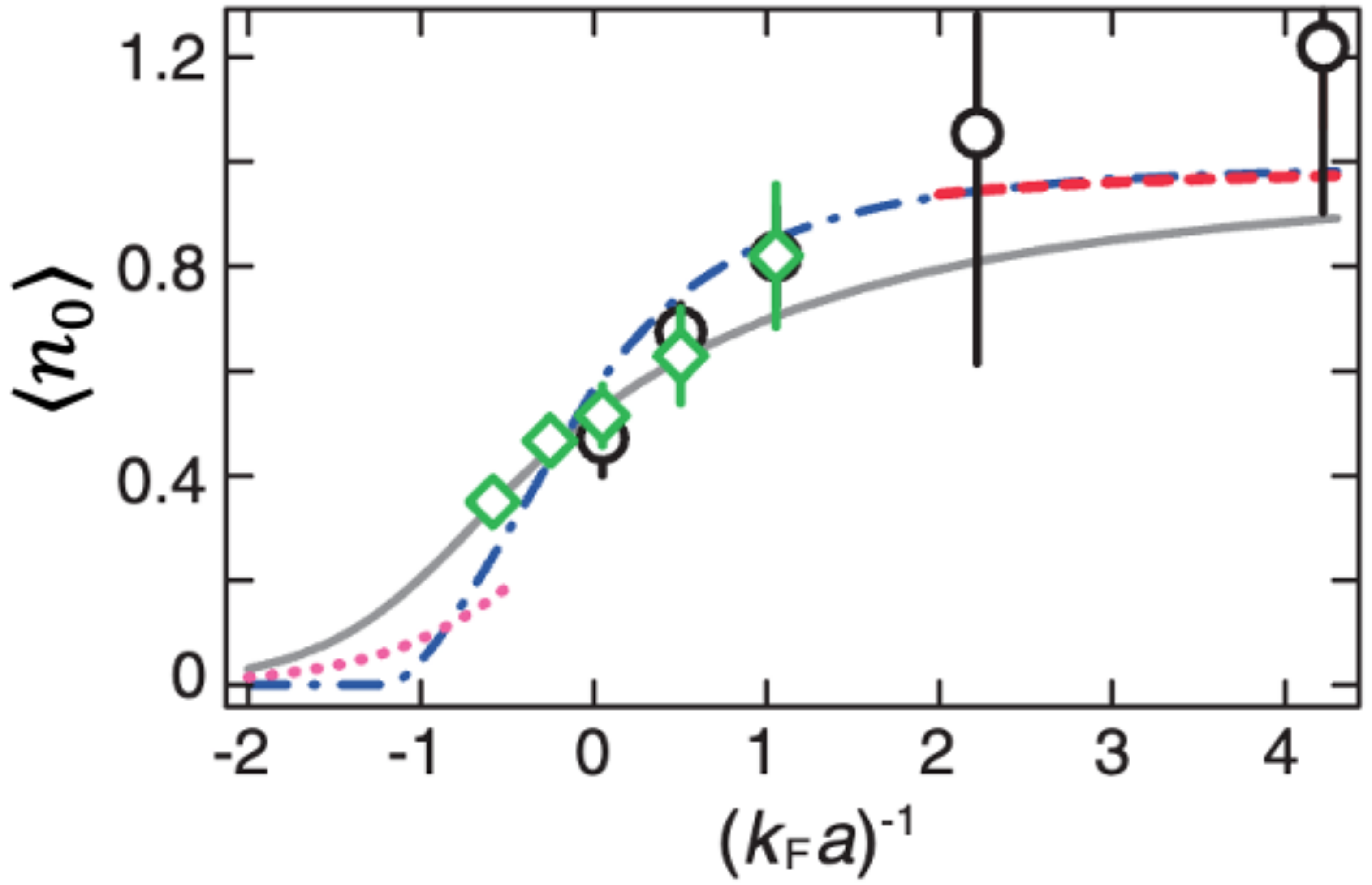
Similarly to the Ambegaokar-Baratoff relation $I_c \sim \Delta$

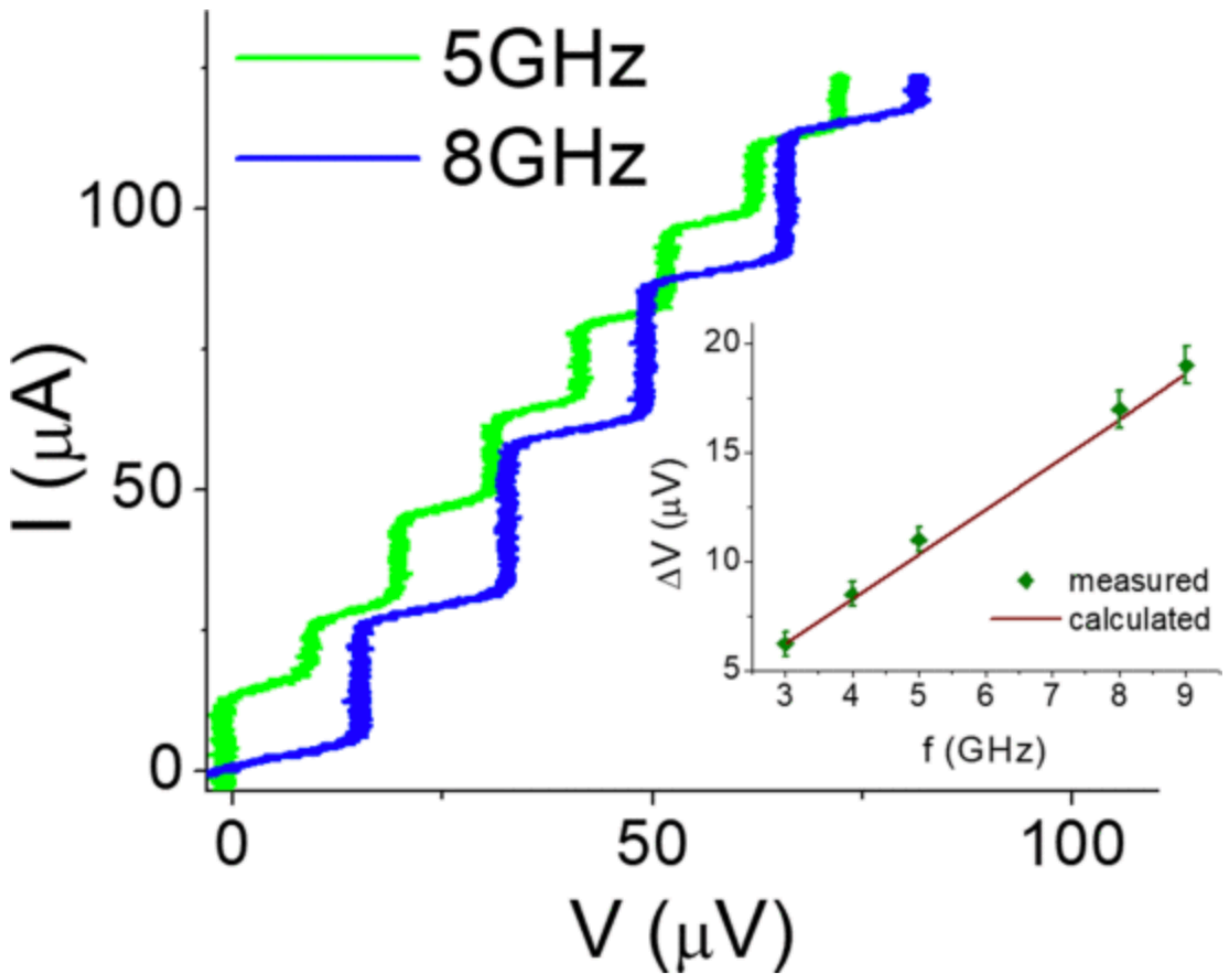
We use the **DC Josephson** effect to extract the condensed fraction of crossover Fermi gases.

☑ Direct measurement of the **order parameter** of strongly-interacting Fermi gases



AC DRIVE OF THE JUNCTION

Shapiro steps in superconducting JJ illuminated by microwaves



R. Caruso, *IEEE Transactions on Applied Superconductivity* 28.7 (2018): 1-6.

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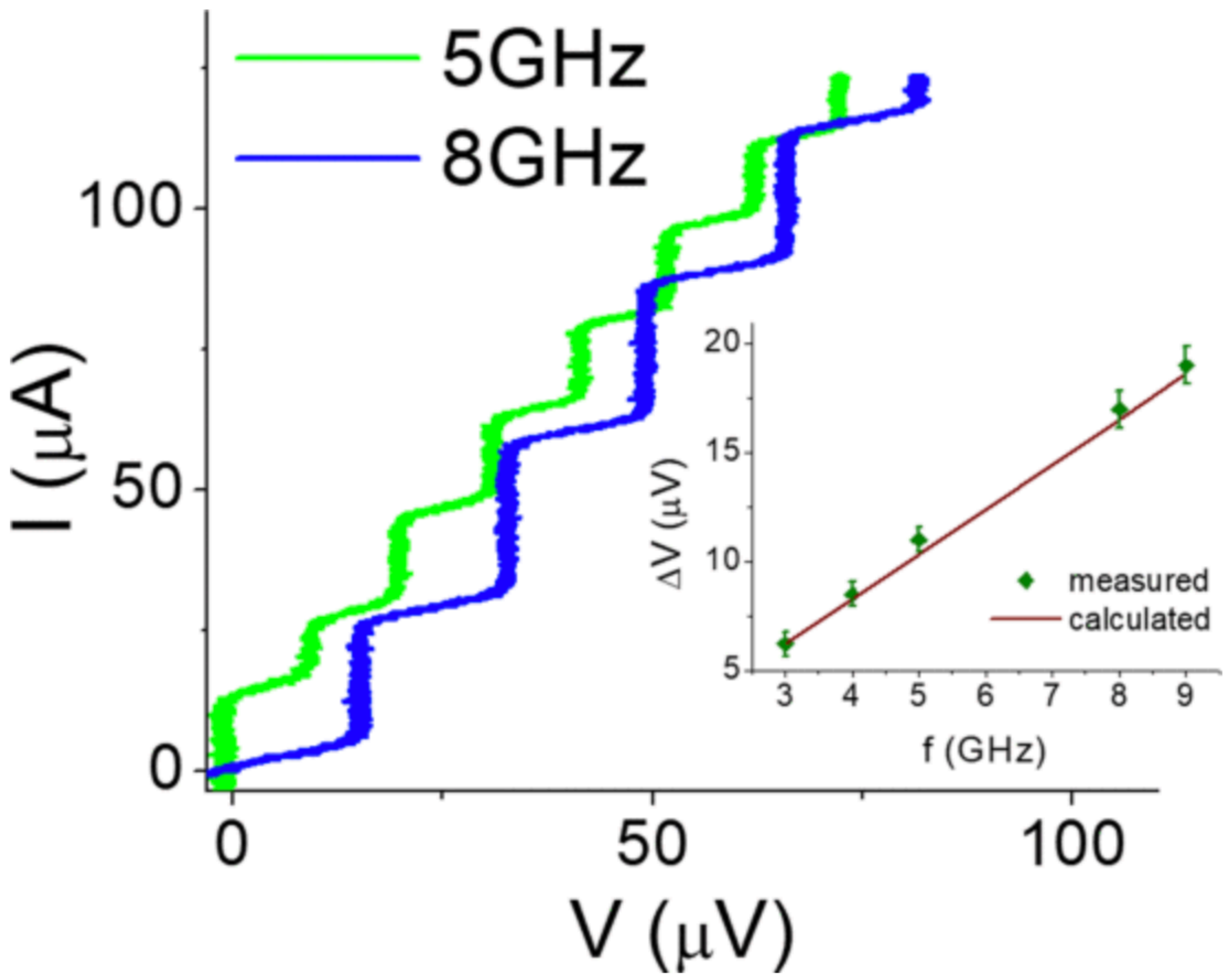

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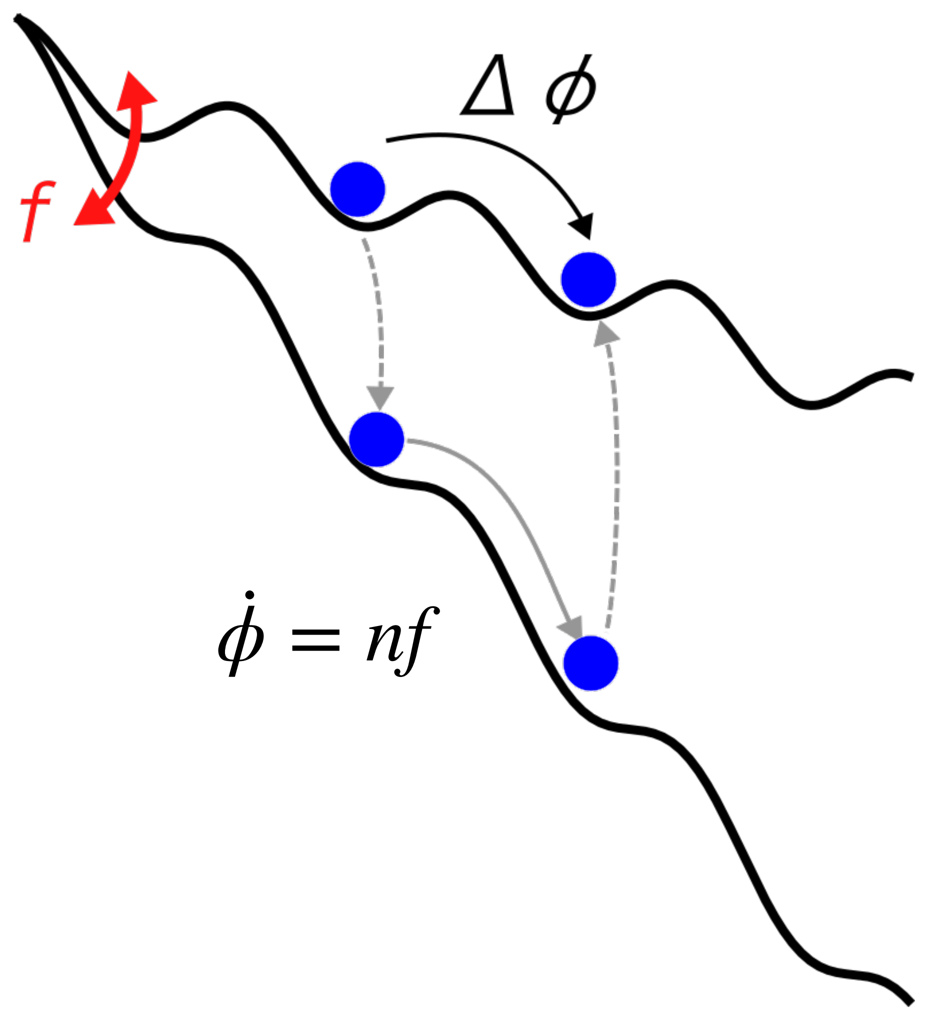
Washboard potential analogy



$$U(\phi) = -I_{ext}(t)\,\phi + I_c\cos(\phi)$$

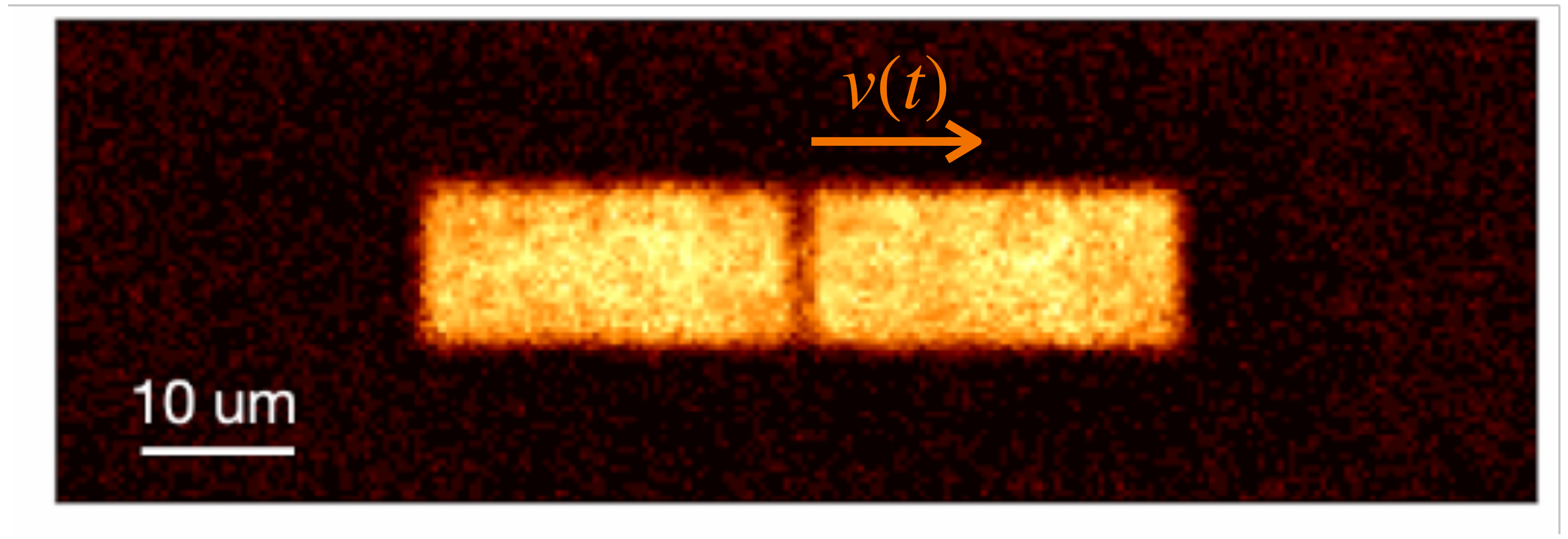
$$I_{ext}(t) = I_0 + I_1\cos(2\pi f t)$$

The phase particle can jump by n minima over one modulation period, when its velocity is **resonant** with a multiple of the driving frequency. When this happen, the phase jump by $2\pi\, n$ every modulation period.

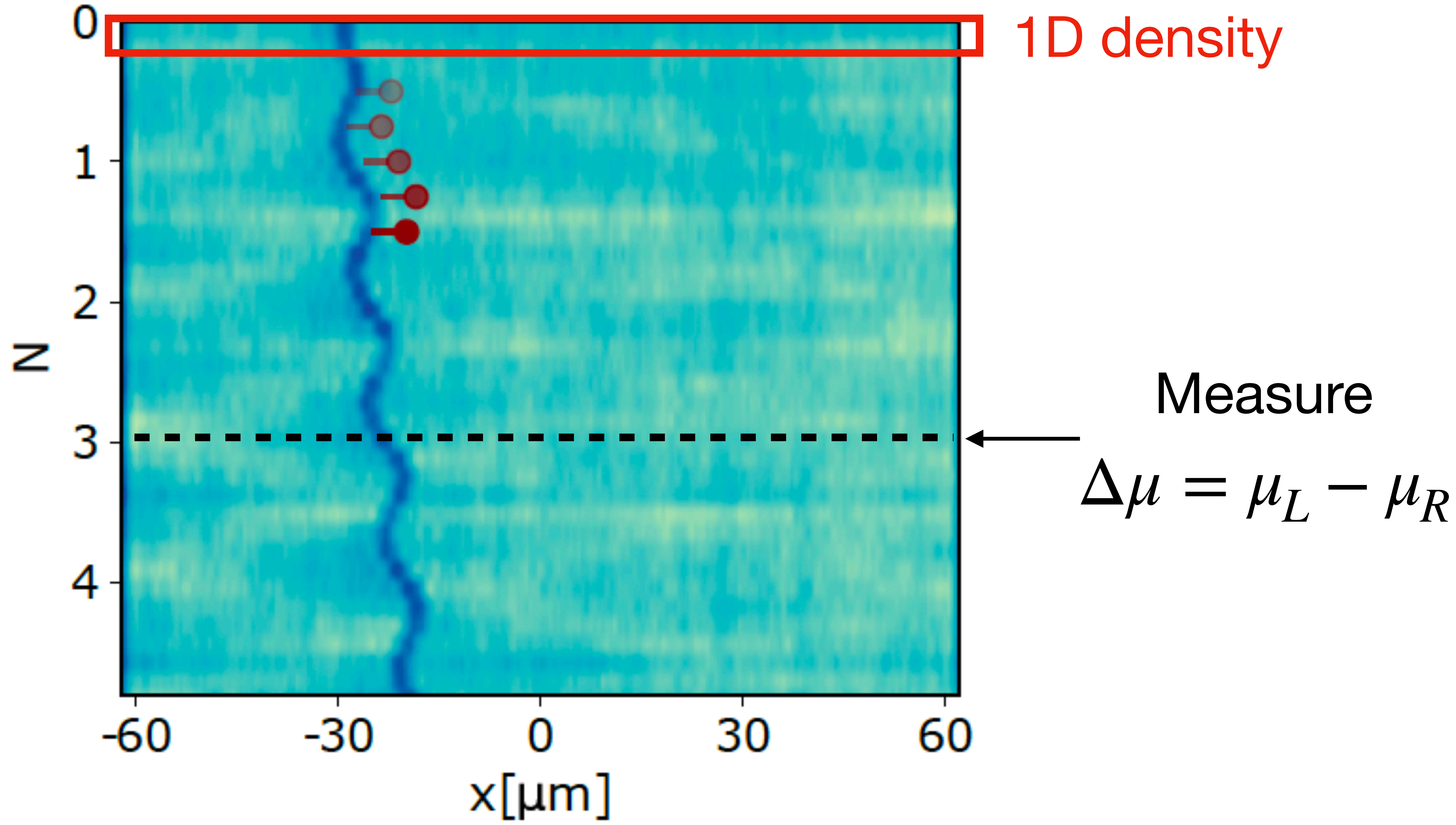
- ☐ Shapiro steps are the results of the **synchronisation** of the phase particle velocity, i.e. $\Delta\mu$, with the external driving frequency $\rightarrow \Delta\mu \sim f$
- ☐ In the $n-$th step the phase undergoes to n phase-slippage process over each modulation period $\rightarrow N_v \sim n$

SHAPIRO STEPS IN ATOMIC FERMI JUNCTIONS

G. Del Pace et al. *Science* **390**, 1125-1129 (2025).



$$v(t) = v_{DC} + v_{AC}\cos(\omega t) \quad \rightarrow \quad I(t) = I_{DC} + I_{AC}\cos(\omega t)$$



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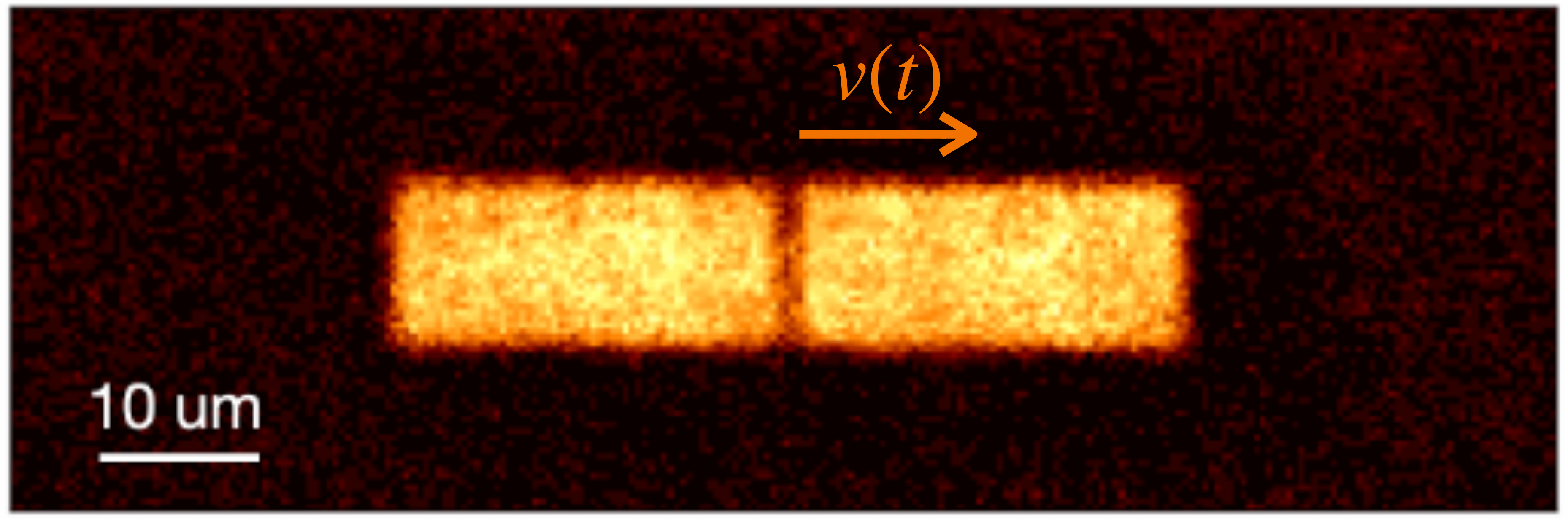

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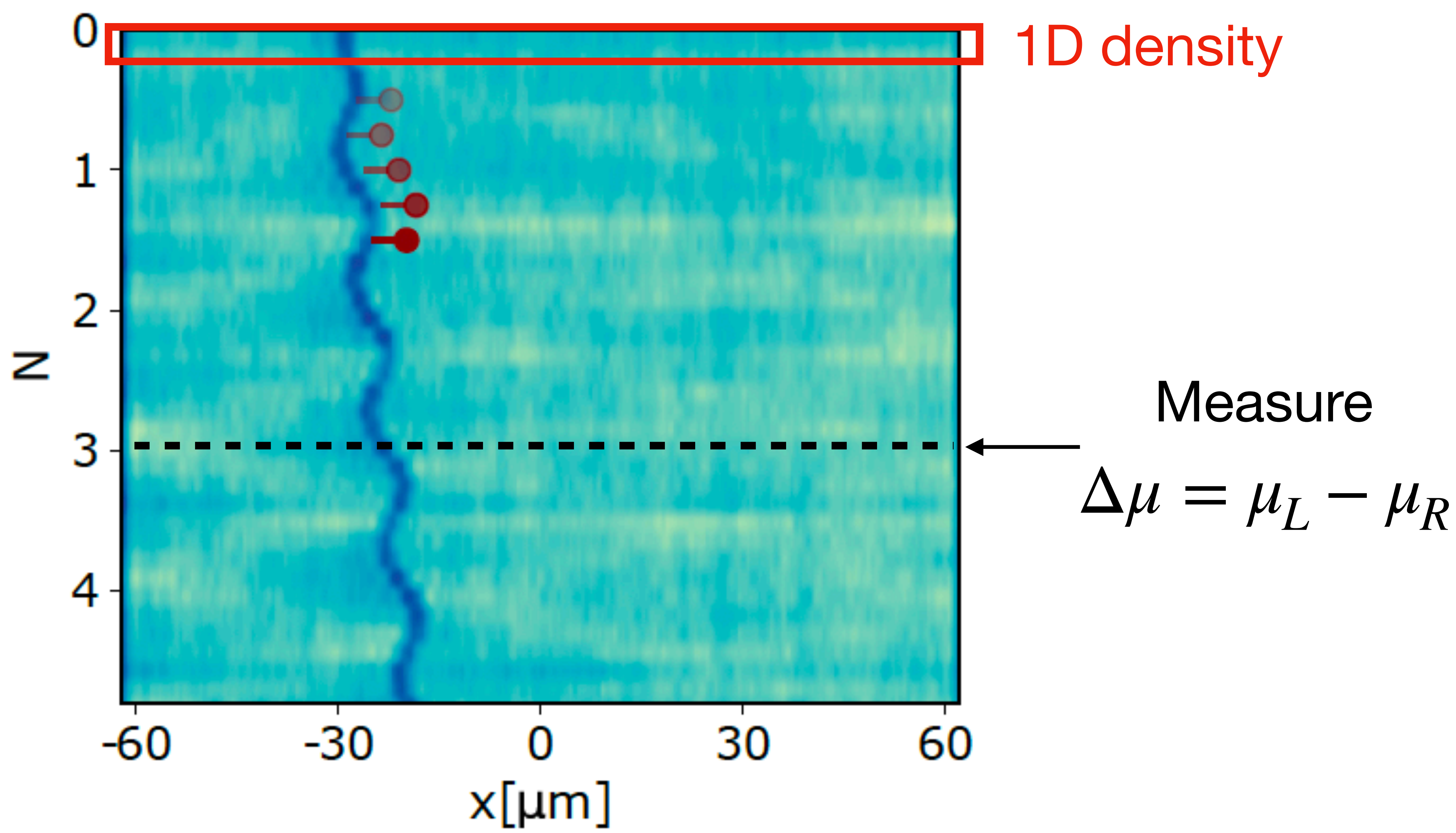
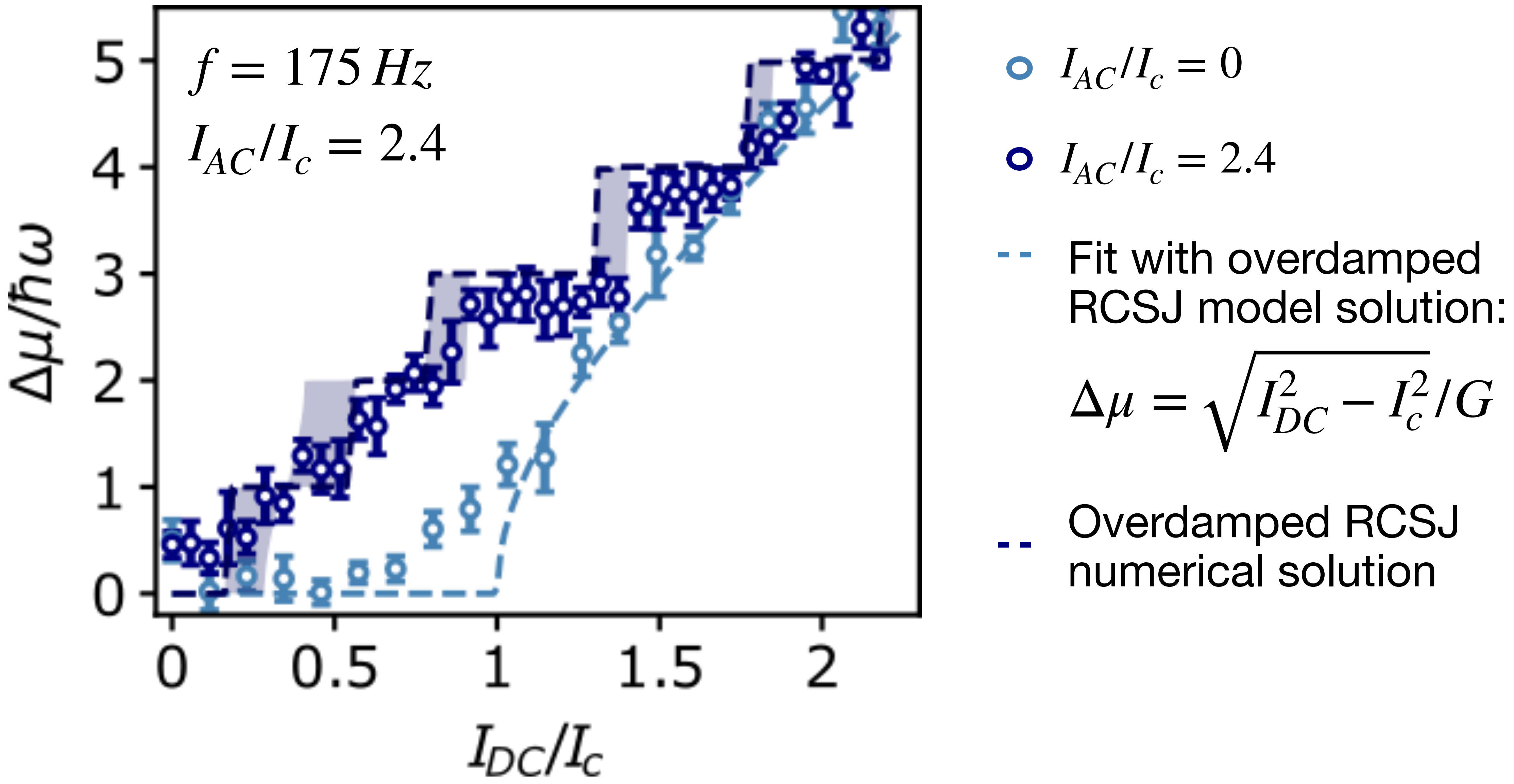


$I - \Delta\mu$ characteristic



Shapiro steps in weakly interacting BECs:
E. Bernhart et al. *Science* **390**, 1130-1133 (2025).

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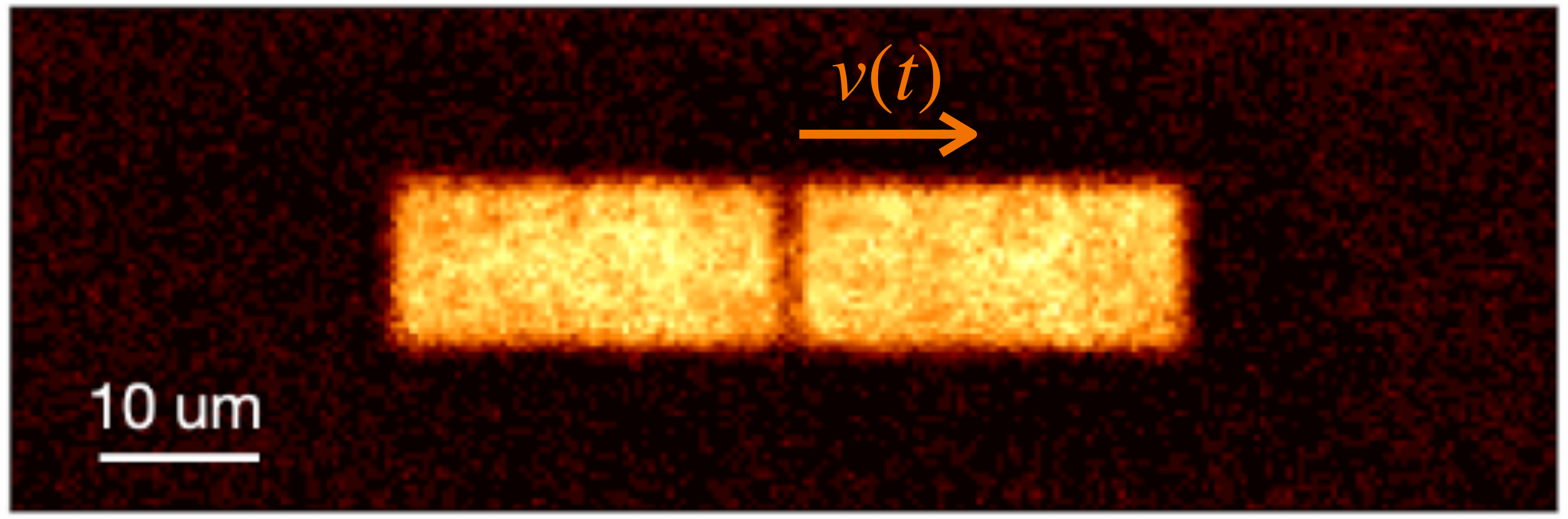

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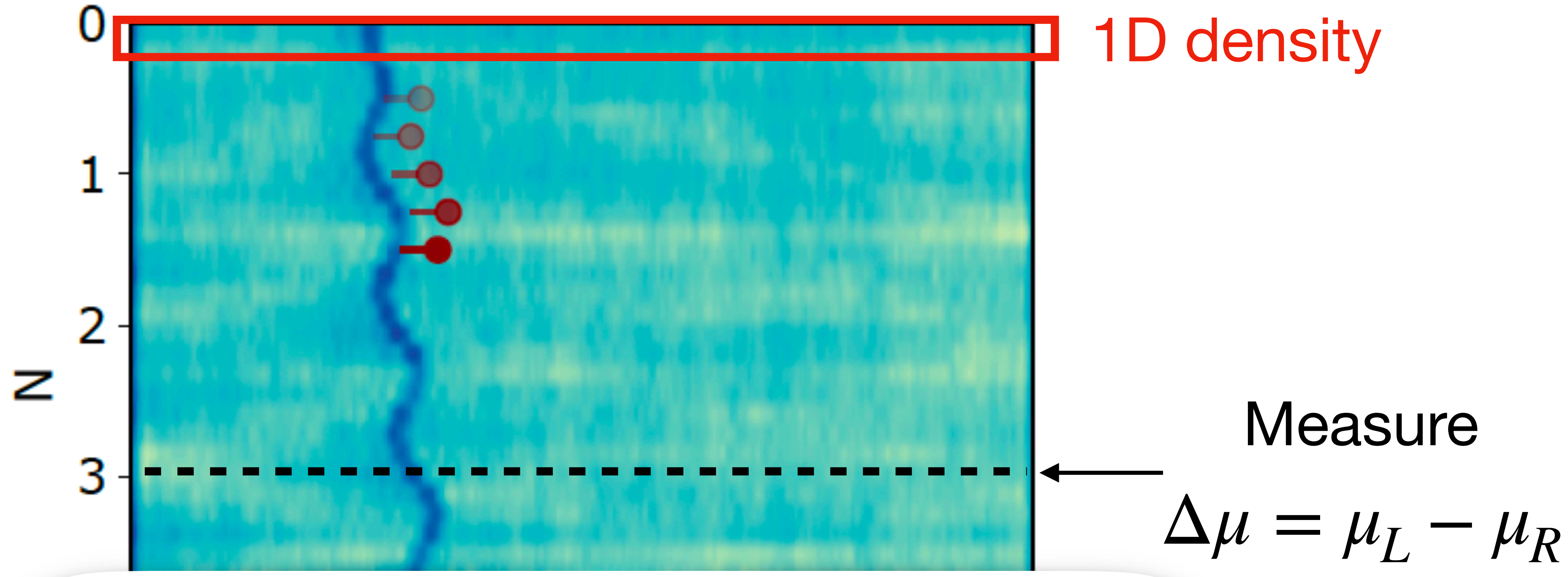

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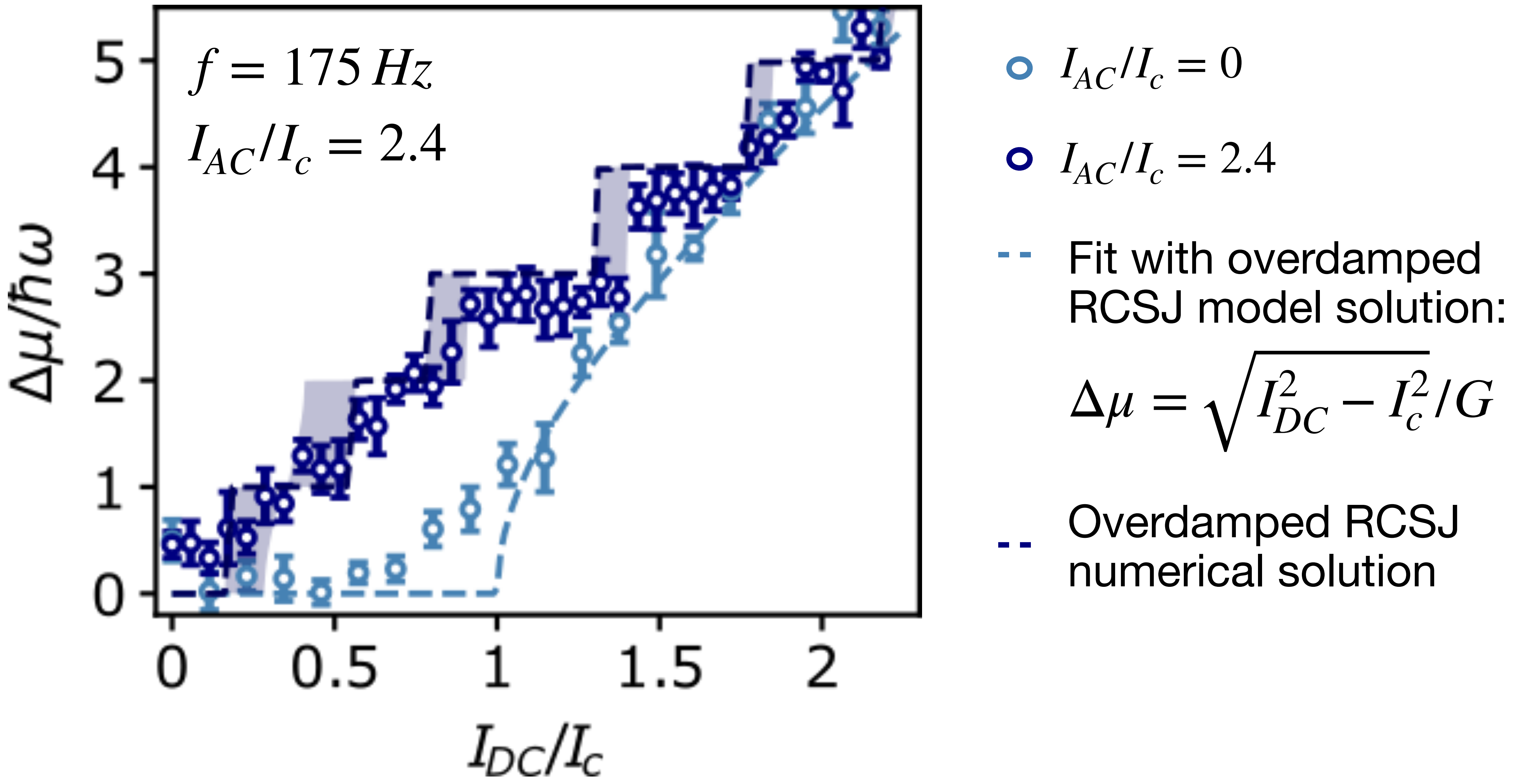
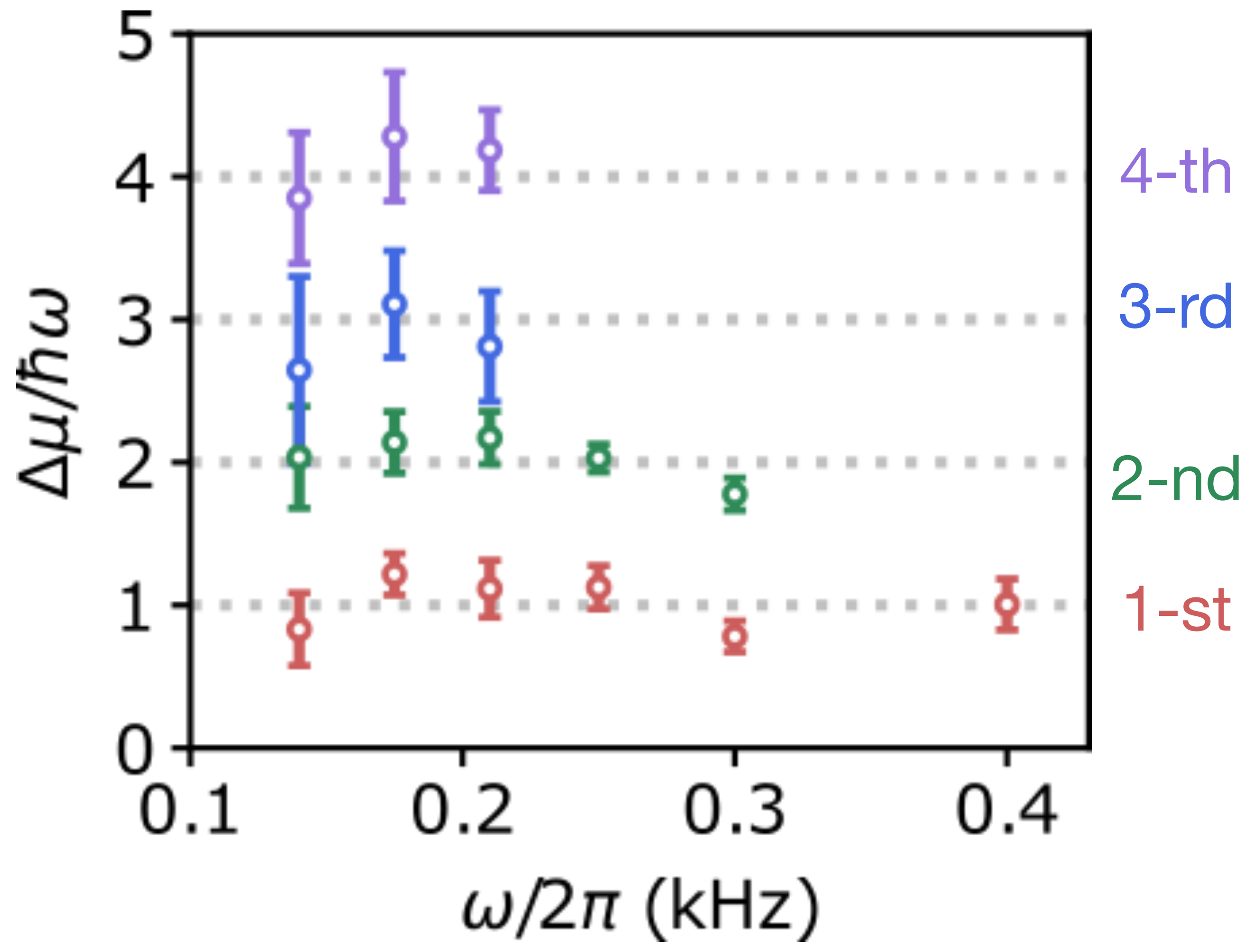


Step height



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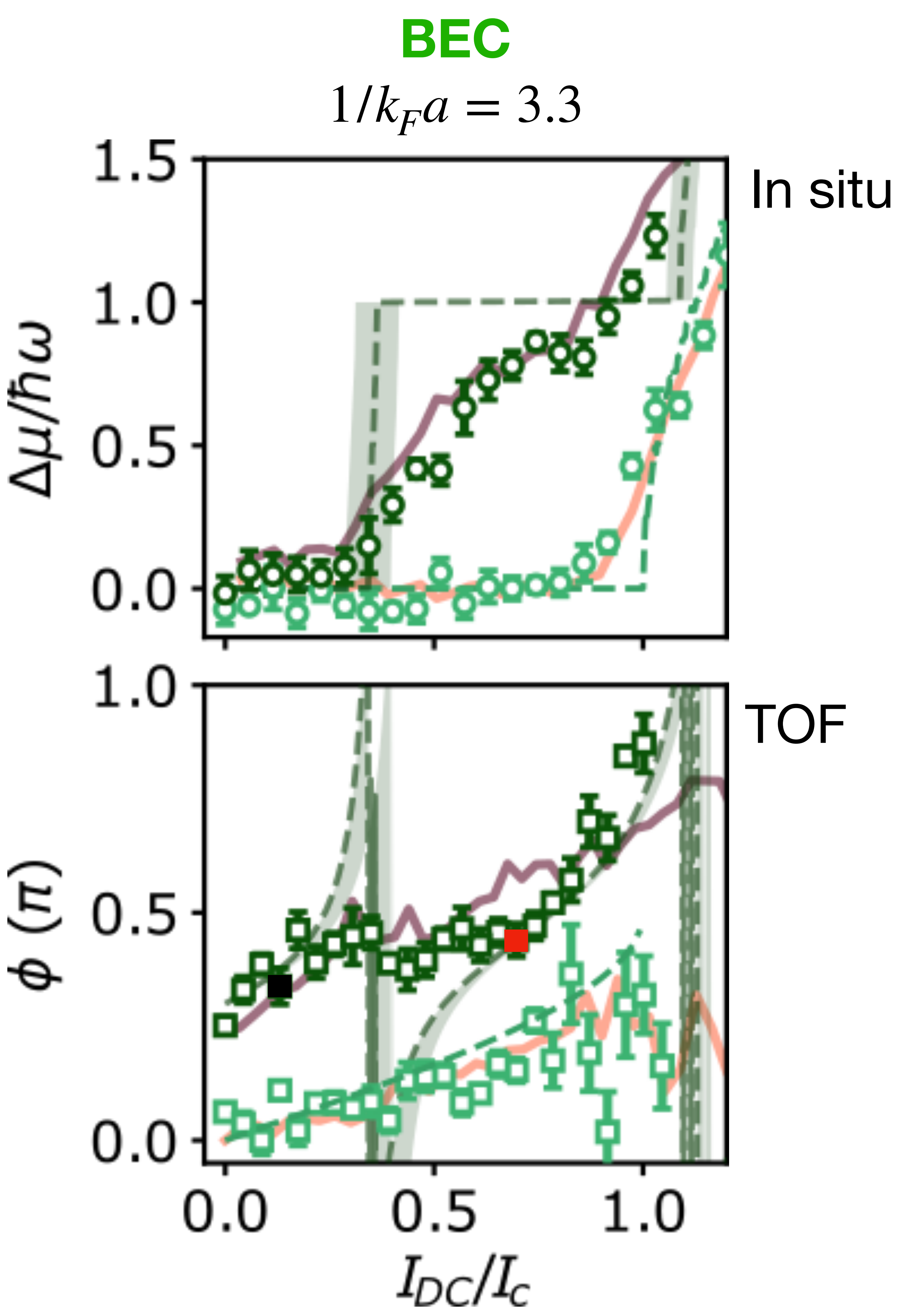
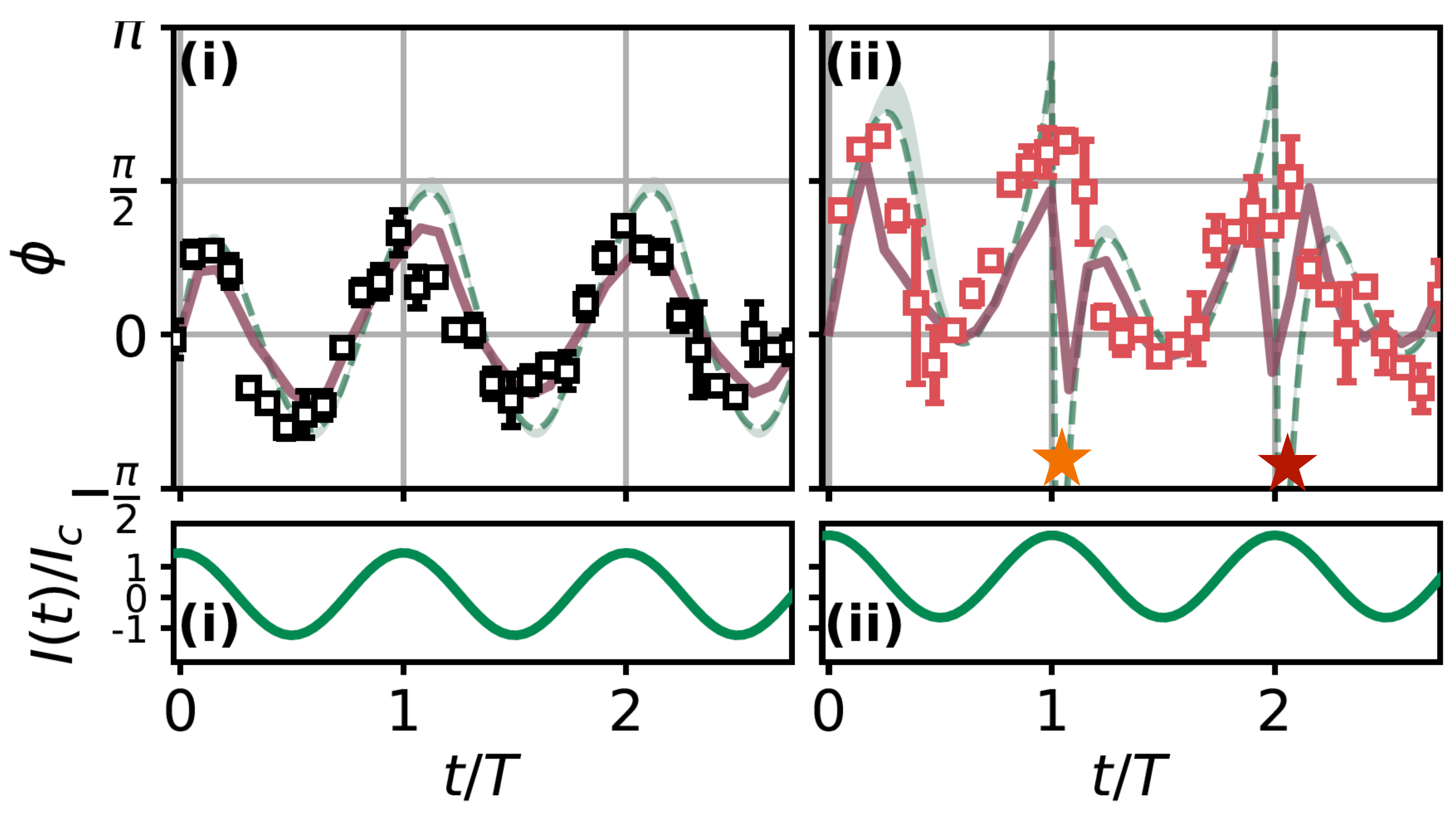
PHASE DYNAMICS AND SYNCHRONIZATION

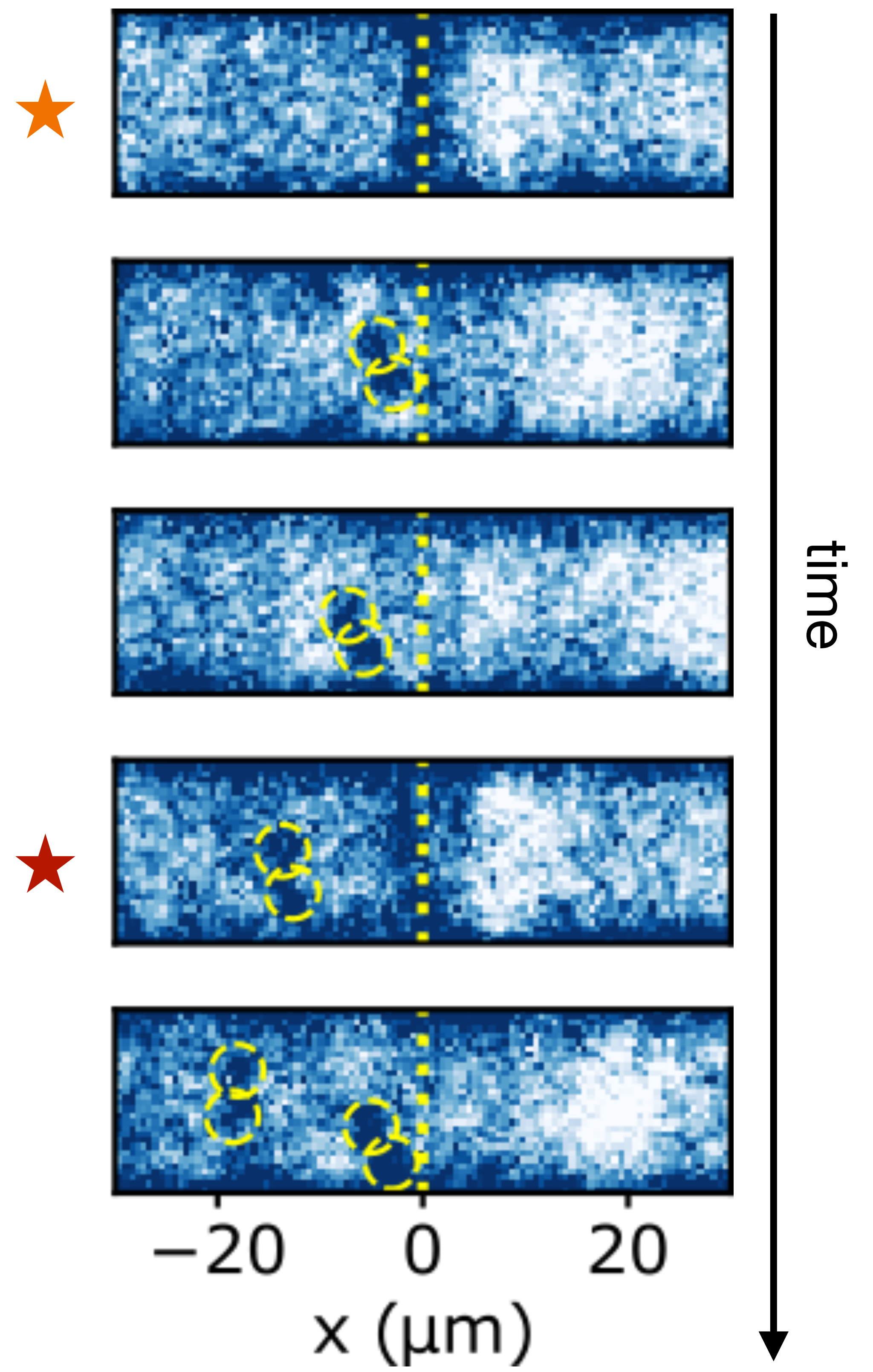
G. Del Pace et al. *Science* **390**, 1125-1129 (2025).

Atomic Josephson junction allow to access directly the relative phase at the junction via time-of-flight (TOF) measurements



Phase time evolution: synchronization and phase slippage

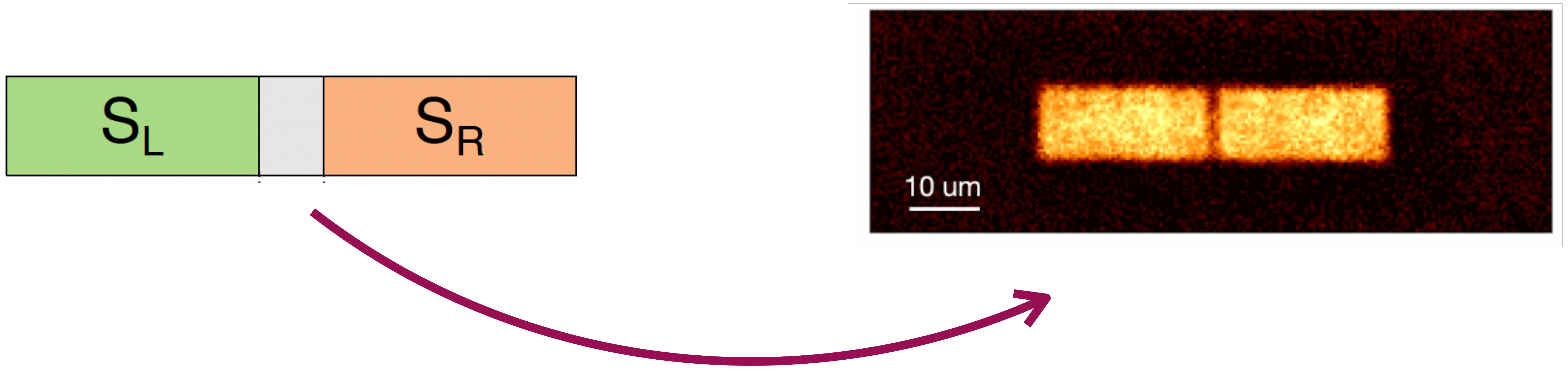




The phase is **synchronized** with the external drive both in the 0-th and n the 1-st step. In the latter it overcomes each modulation cycle the threshold for **phase slippage**, which manifest in our geometry as vortex-anti vortex pairs.

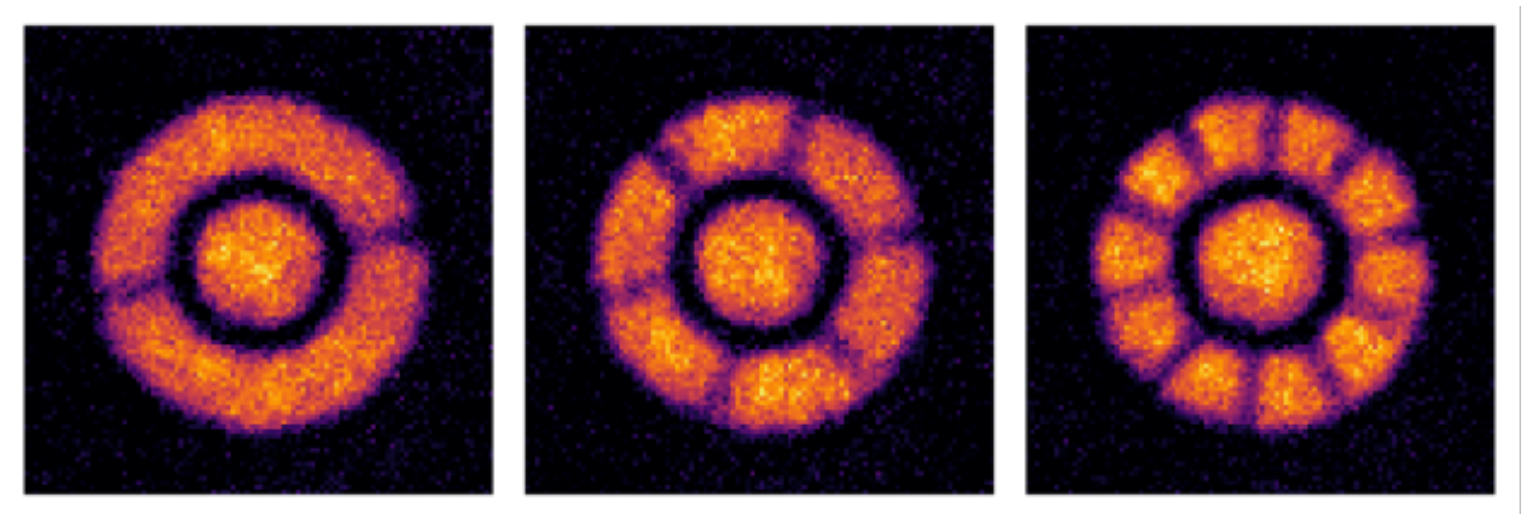
Dynamical classical-filed numerical simulation under similar experimental conditions by V. P. Singh (TII, Abu Dhabu)

CONCLUSIONS



Quantum simulation

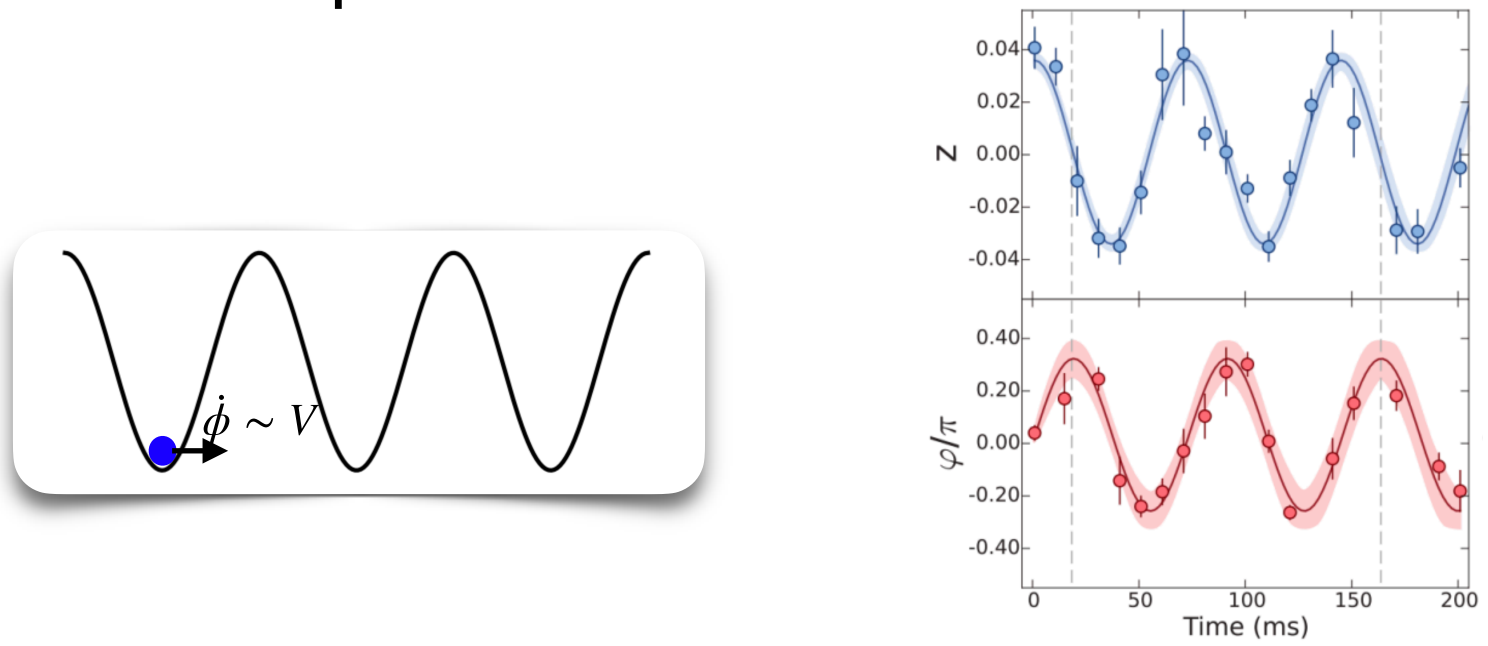
- Understand a complex system via the use of a second clean one: Fermi superfluid as **quantum simulator** of condensed matter
- Use atomic JJs as building blocks for more complex architecture: **atomtronics**



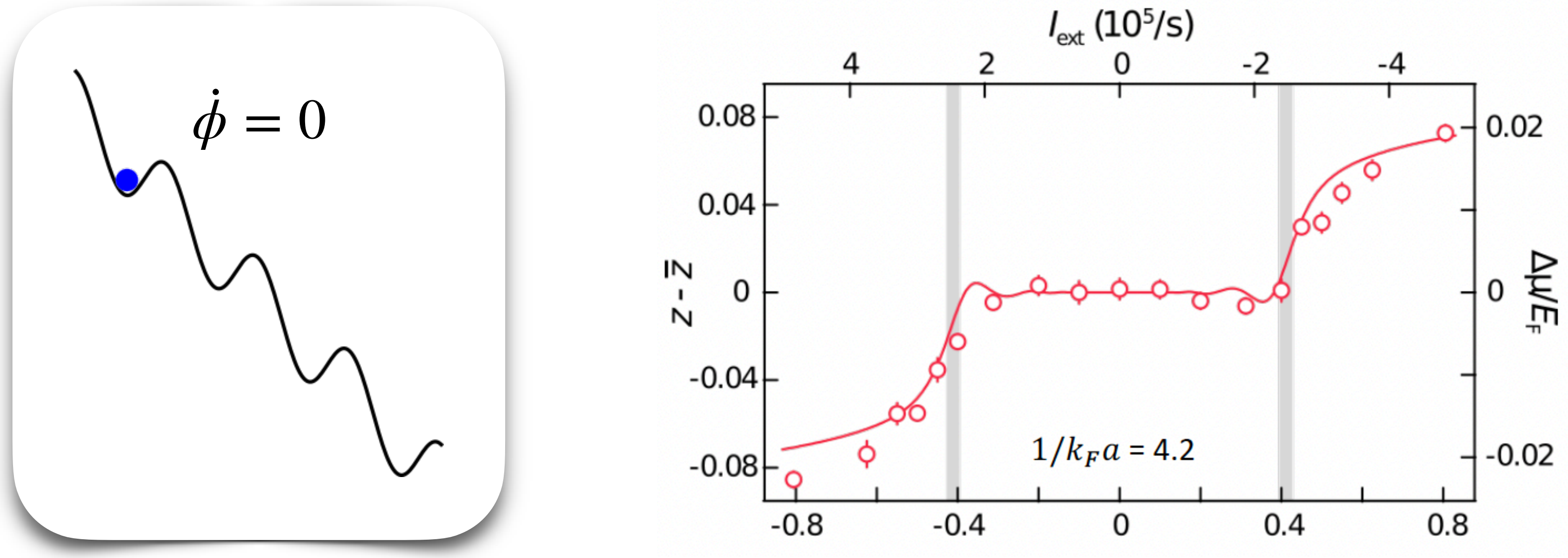
L. Pezzè, et al. *Nat. Comm.* 15.1: 4831 (2024)

[1] G. Valtolina et al., *Science* **350**.6267 (2015)
[2] W. J. Kwon, et al., *Science*, **369**, 6499 (2020)
[3] G. Del Pace et al. *Science* **390**, 1125-1129 (2025).

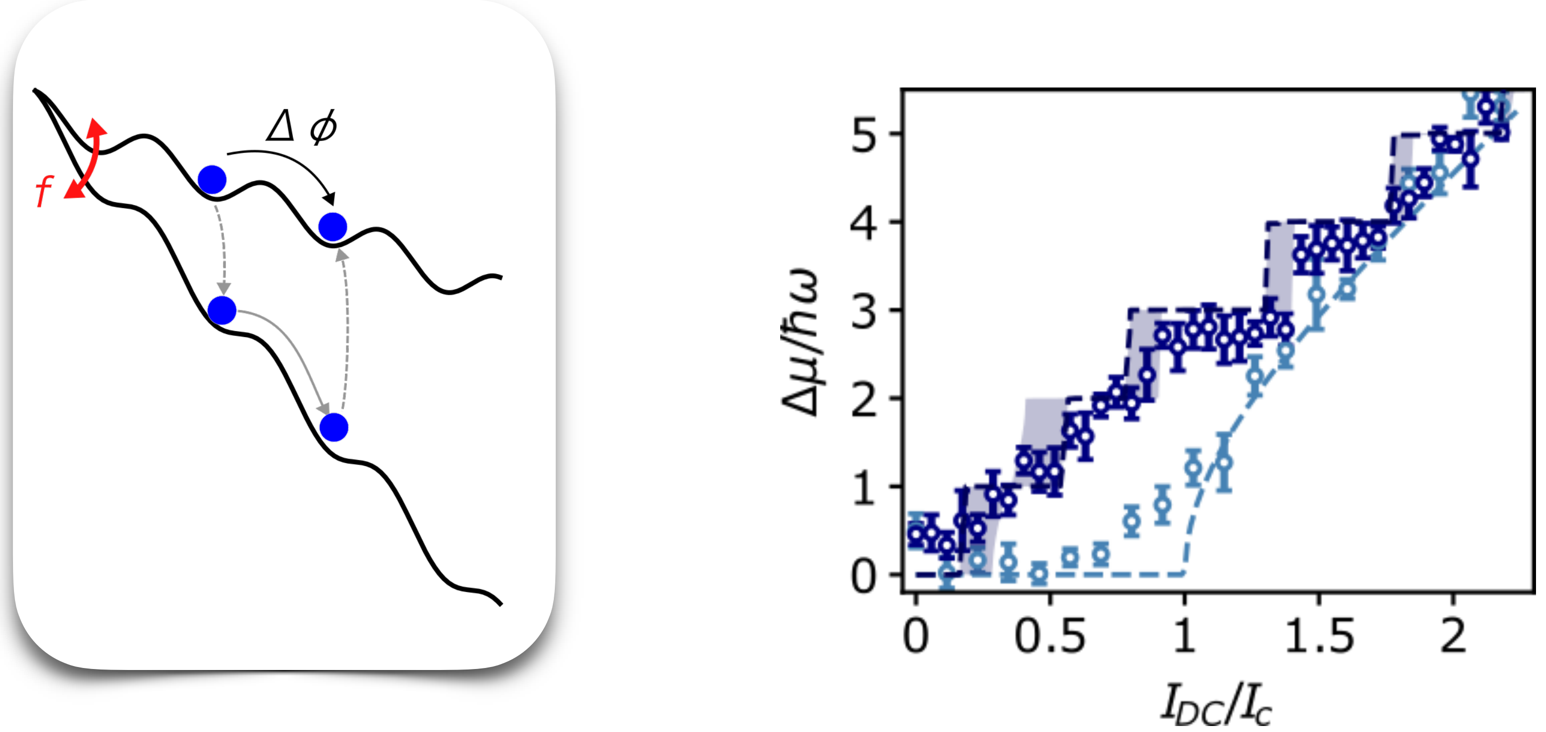
- AC Josephson effect [1]



- DC Josephson effect [2]



- Shapiro steps under AC drive [3]



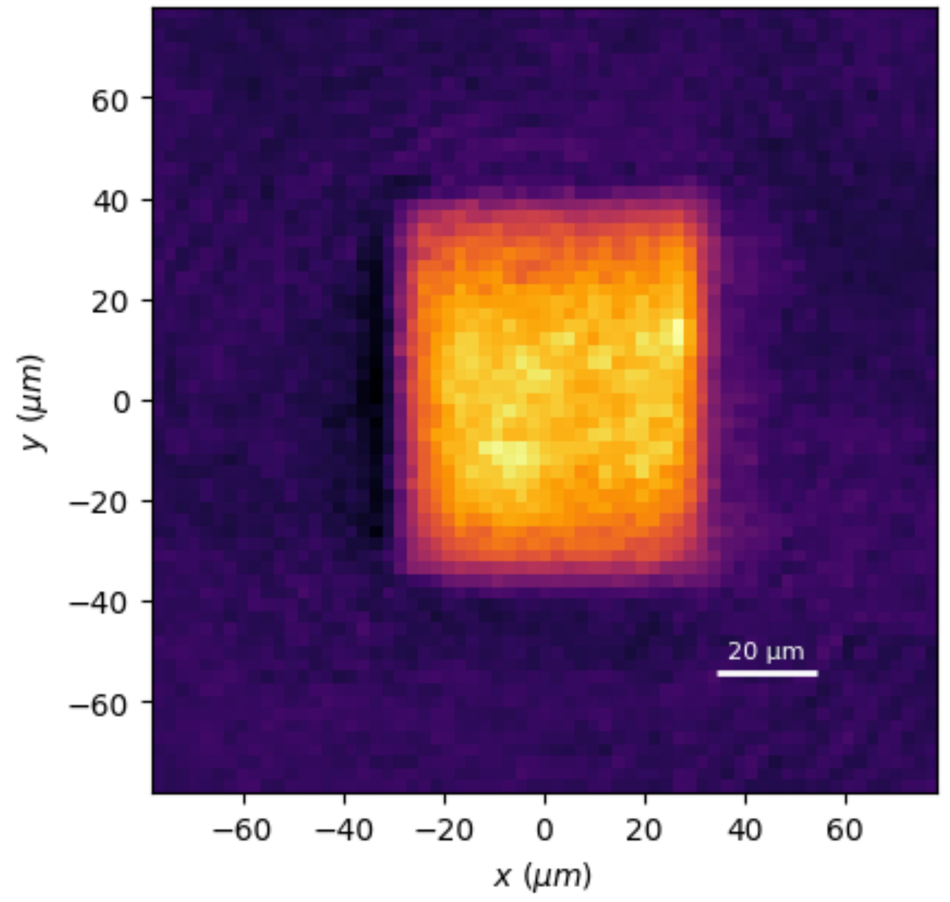
CURRENTLY IN THE LAB

Homogeneous box potential

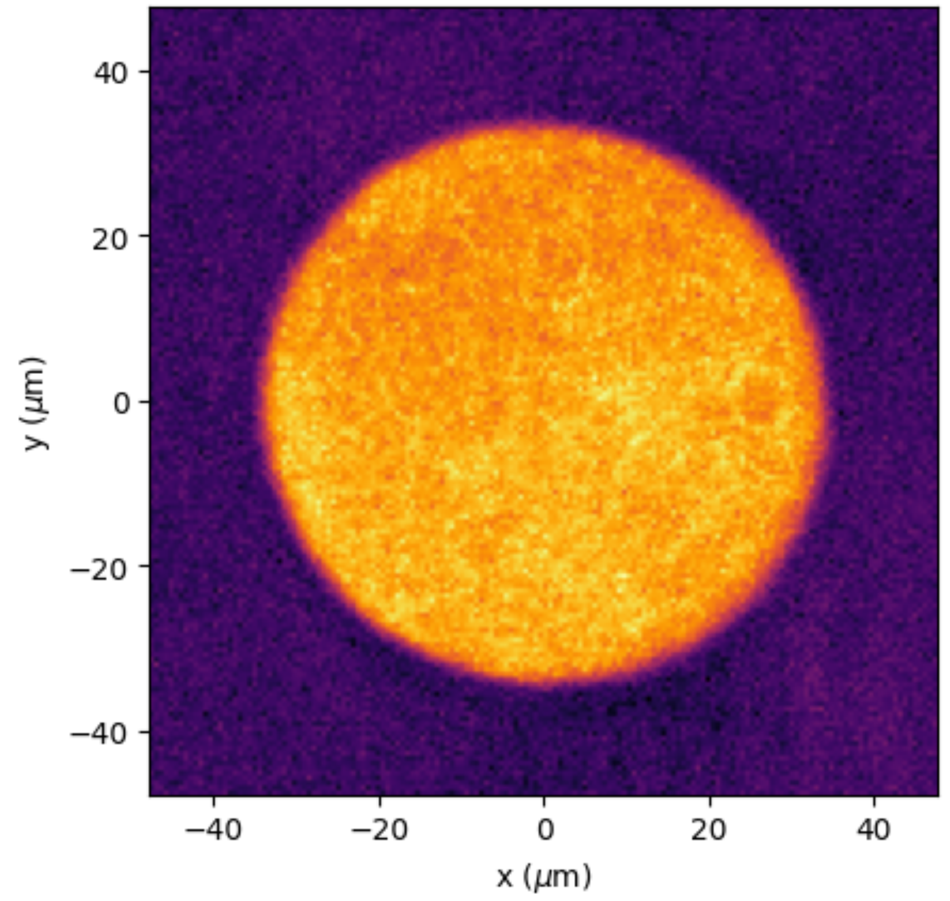
- Second DMD setup along the horizontal direction to have tunable homogeneous potential.
- Magnetic levitation to compensate for gravity.

☑ Ultracold Fermi gas with homogeneous density

Horizontal imaging



Vertical Imaging



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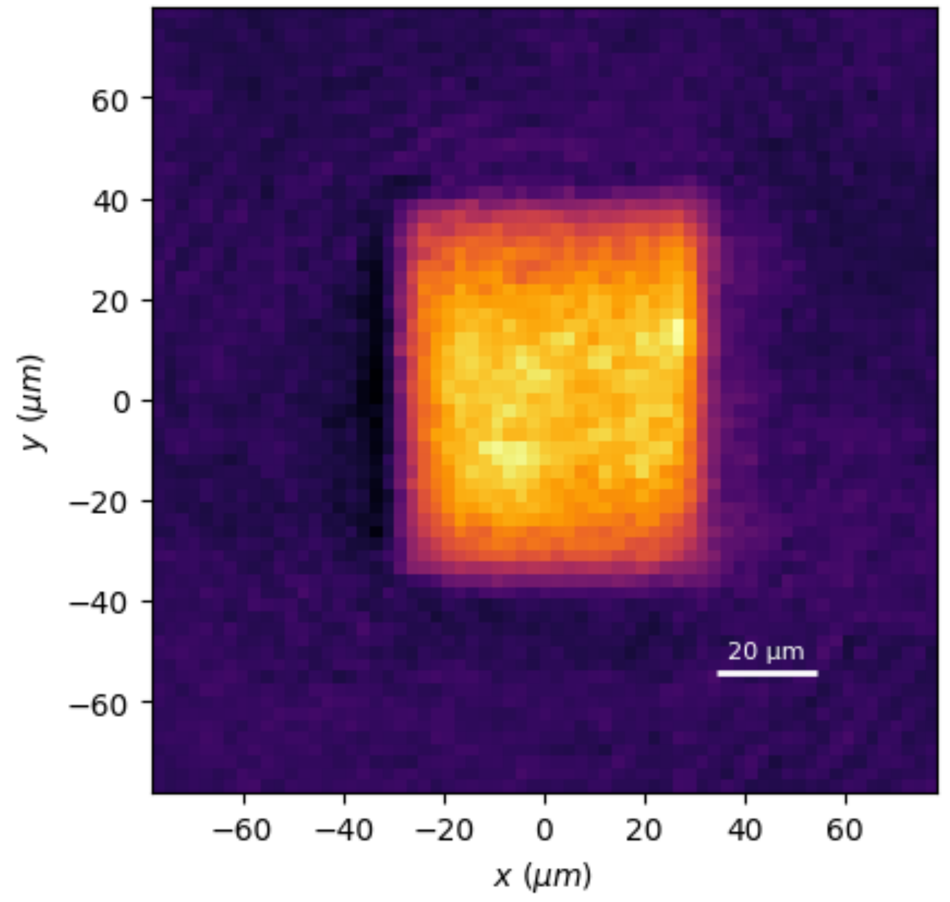
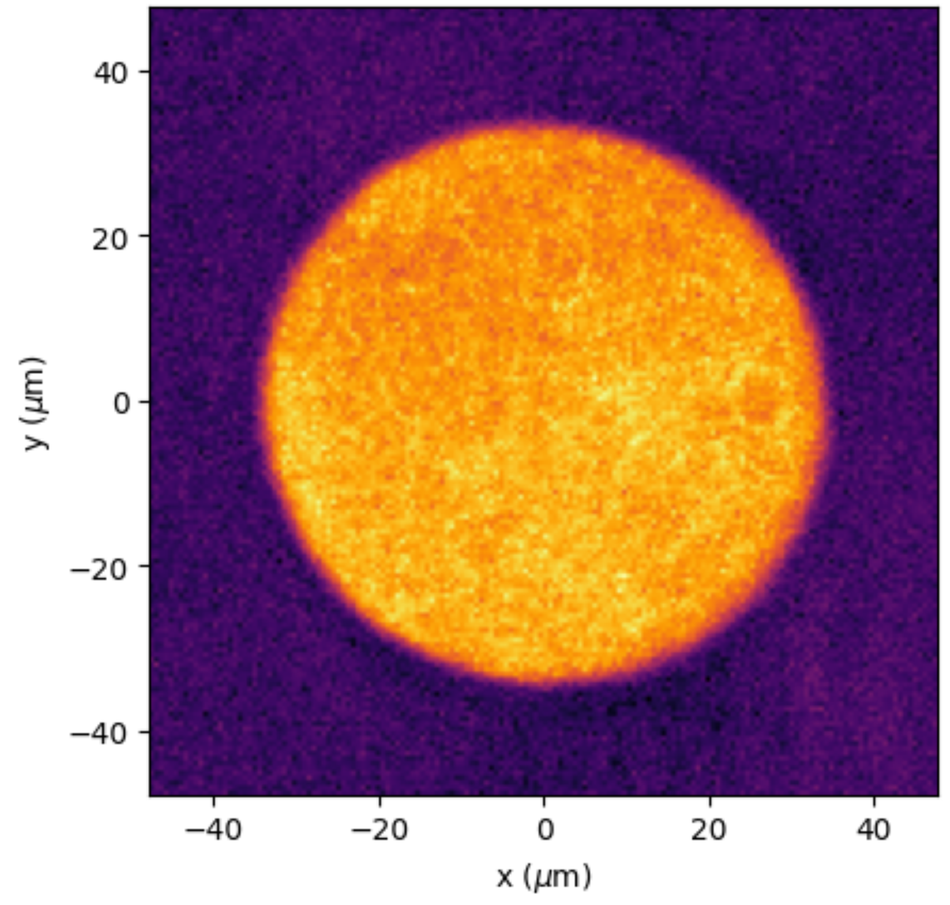


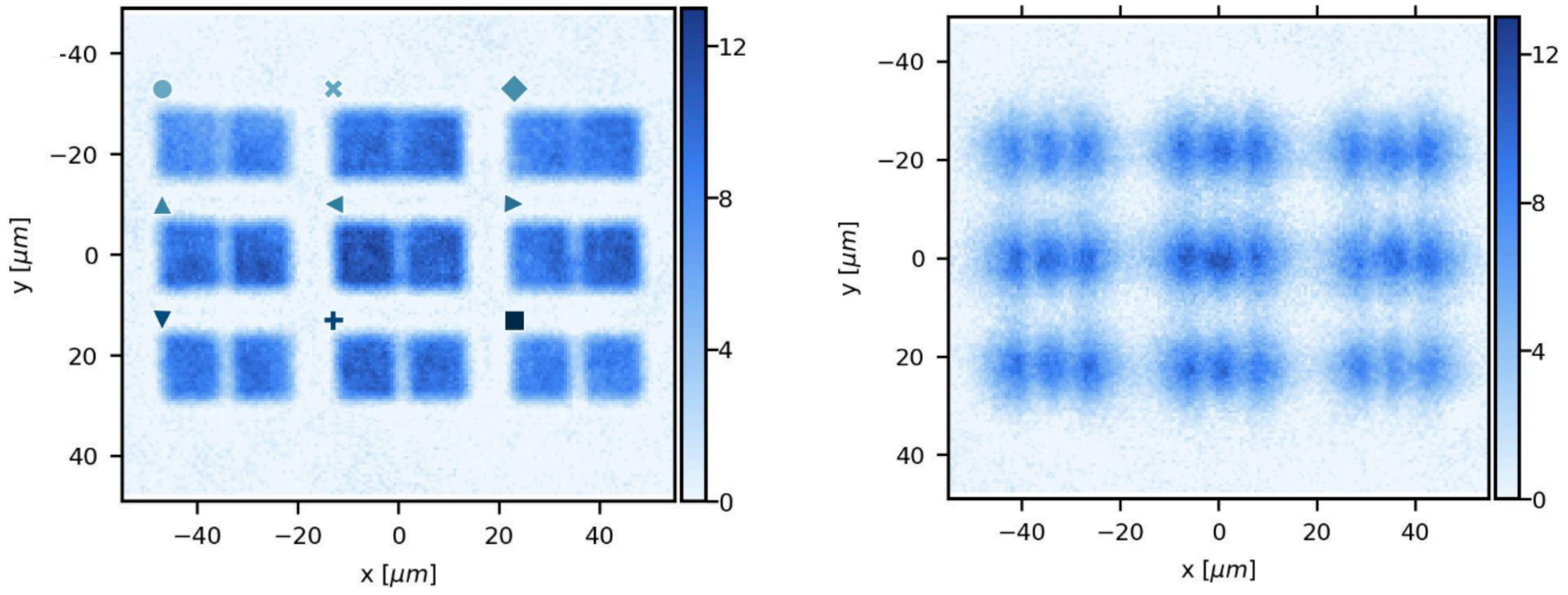
Vertical Imaging



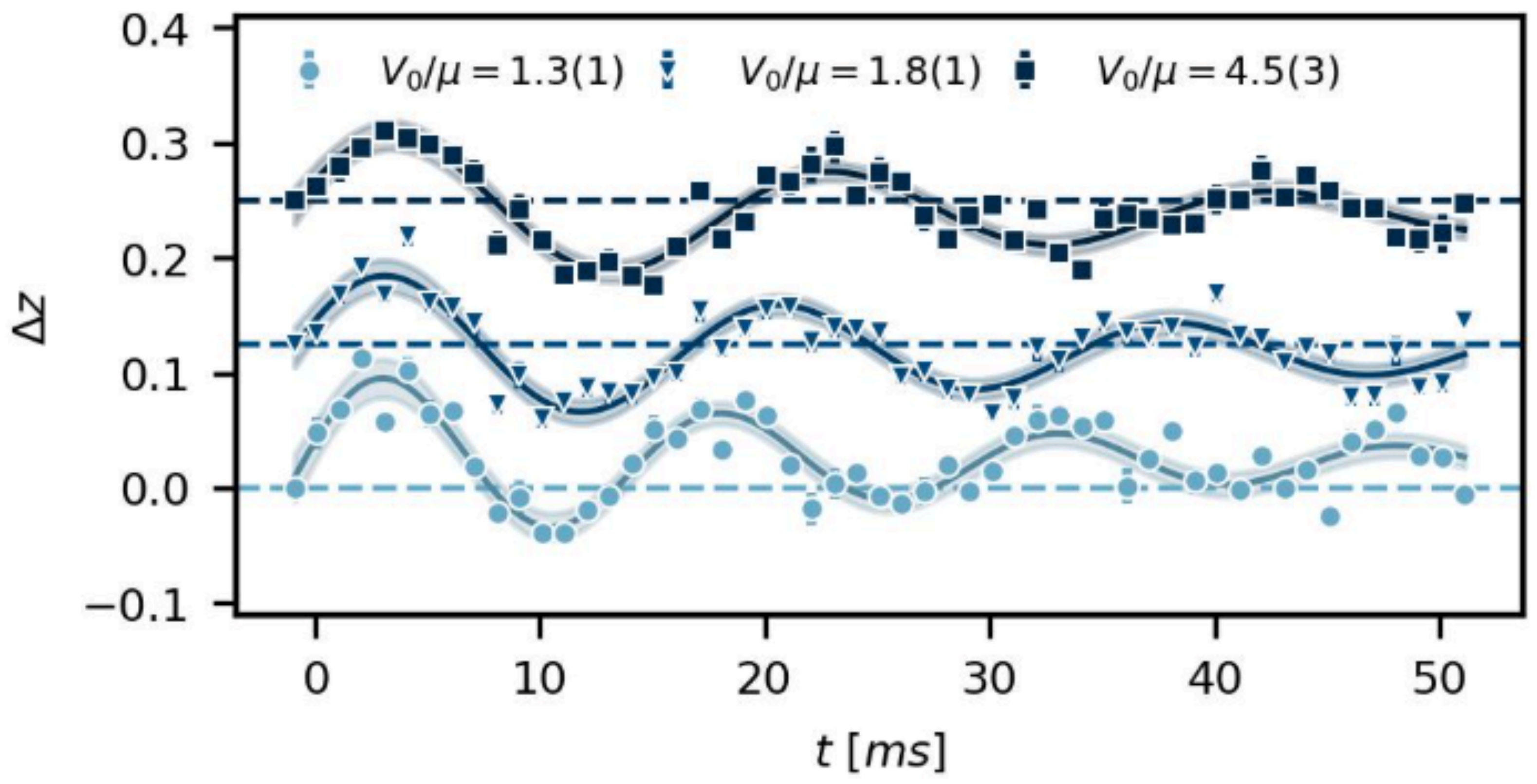
Multiplexing quantum simulation

D. Hernández-Rajkov, *in preparation*

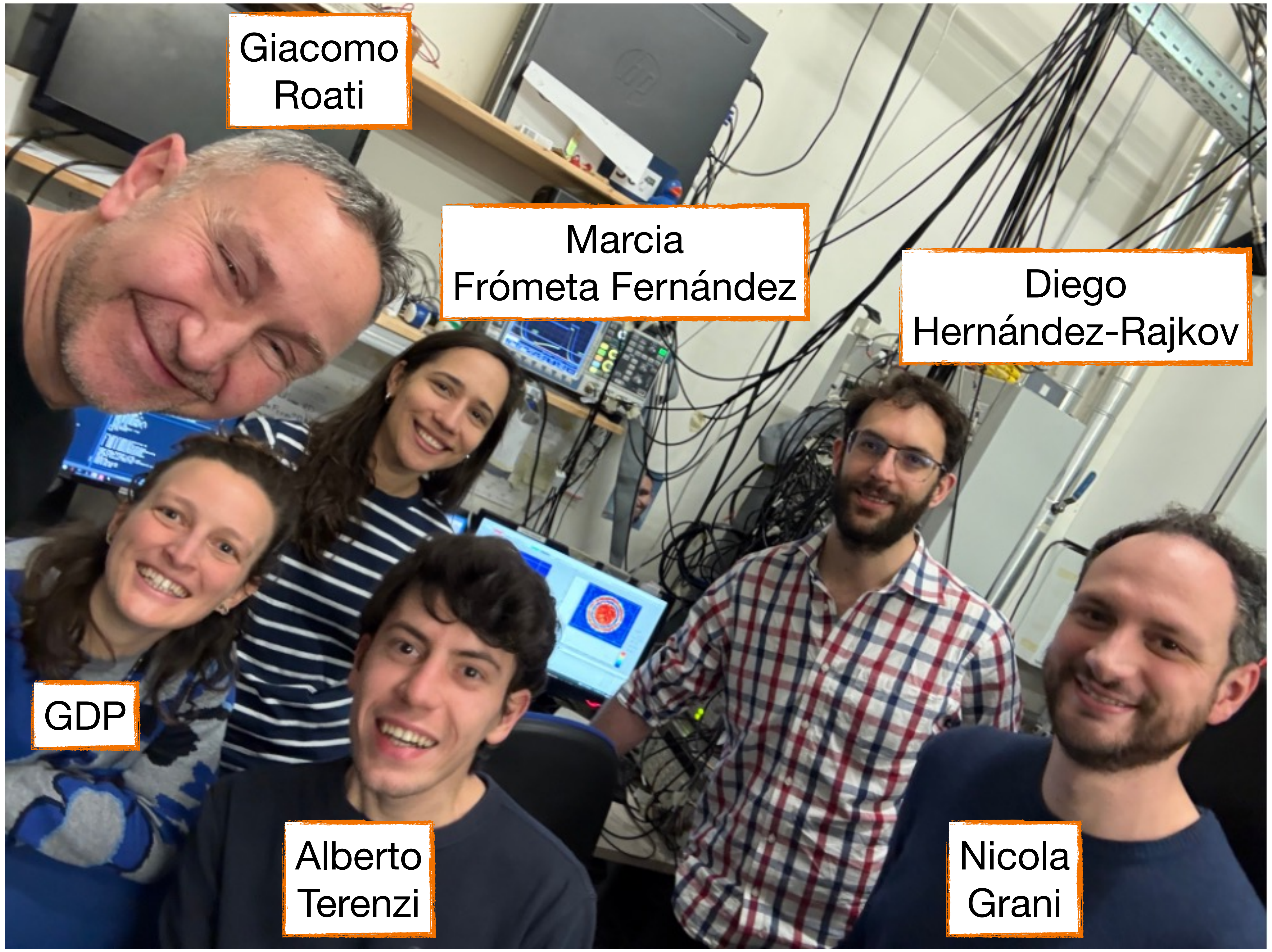
- Parallel operation of different junctions, individually addressable



- Follow the Josephson dynamic of different junctions simultaneously



LITHIUM TEAM @ LENS

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Lens European Laboratory for Non-Linear Spectroscopy

PASQuanS2

NQSTI National Quantum Science and Technology Institute

INRiM ISTITUTO NAZIONALE DI RICERCA METROLOGICA

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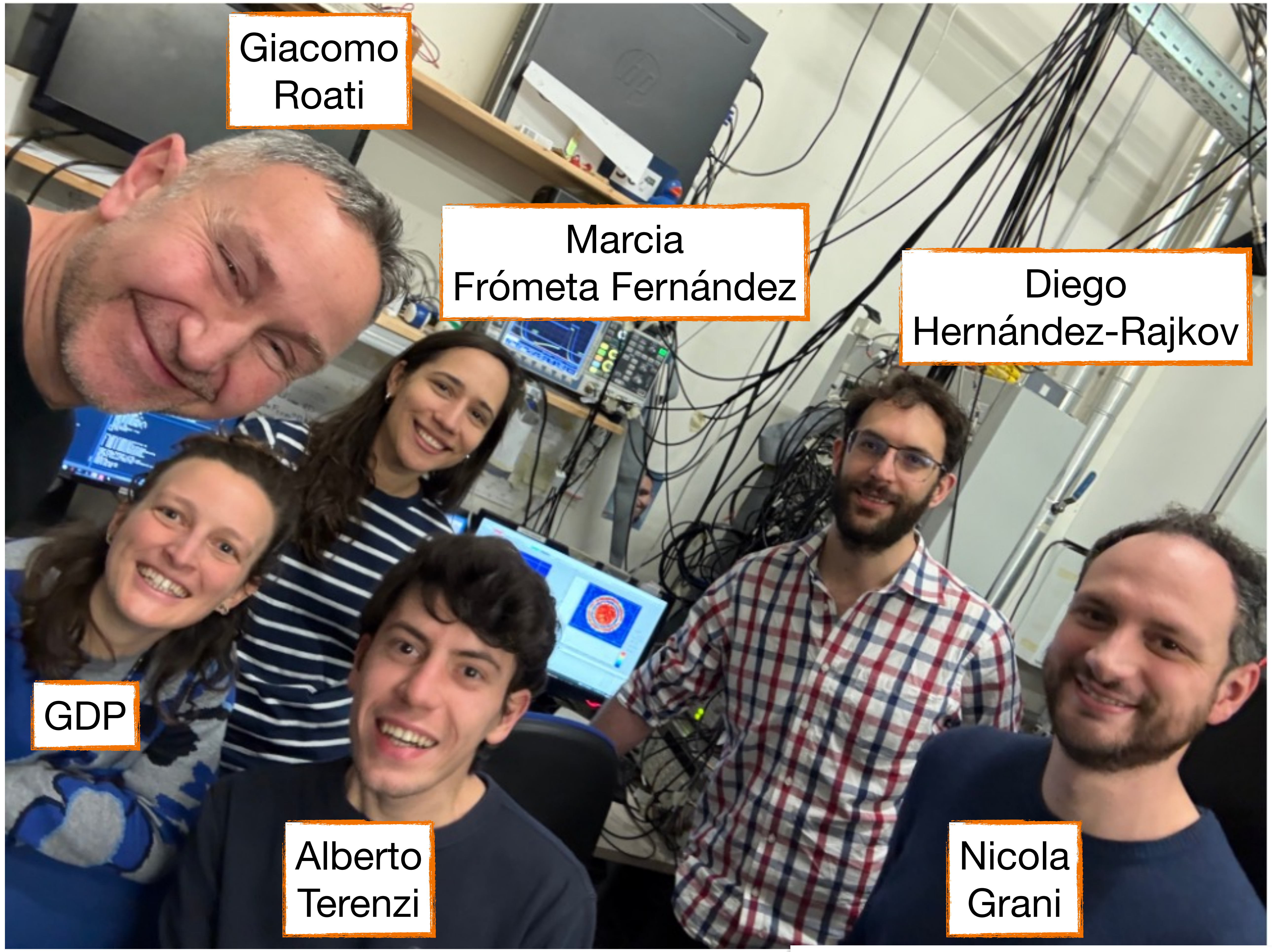

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